The Effects of Cross-Sectional Scale Differences on Regression Results in Empirical Accounting Research*

MARY E. BARTH, *Stanford University*

SANJAY KALLAPUR, *University of Arizona*

Abstract. This study investigates coefficient bias and heteroscedasticity resulting from scale differences in accounting levels-based research designs analytically and using simulations based on accounting data. Findings indicate that including a scale proxy as an independent variable is more effective than deflation at mitigating coefficient bias, even if the proxy is 95 percent correlated with the true scale factor. In fact, deflation can worsen coefficient bias. Also, deflation often does not noticeably reduce heteroscedasticity and can decrease estimation efficiency. White (1980) standard errors are close to the true ones in regressions using undeflated variables. Replications of specifications in three recent accounting studies confirm the simulation findings. The findings suggest that when scale differences are of concern, accounting researchers should include a scale proxy as an independent variable and report inferences based on White standard errors.

Résumé. Les auteurs examinent, tant sur le plan analytique qu'au moyen de simulations basées sur les données comptables, la distorsion des coefficients et l'hétéroscédasticité résultant des différences d'échelle dans les plans de recherche comptable basés sur les niveaux. Leurs constatations révèlent que l'inclusion d'un substitut d'échelle à titre de variable indépendante est plus efficace que la déflation pour atténuer la distorsion relative au coefficient, même si le substitut présente une corrélation de 95 pour cent avec le véritable facteur d'échelle. En fait, la déflation peut accentuer la distorsion relative au coefficient. Aussi, il arrive souvent que la déflation, sans réduire de façon appréciable l'hétéroscédasticité, puisse diminuer l'efficience de l'estimation. Les erreurs-types de White (1980) se rapprochent des erreurs véritables dans les régressions faisant appel à des variables non déflatées. La répétition des mêmes caractéristiques dans trois études comptables récentes confirme les résultats de la simulation. Les conclusions de l'étude donnent à penser que lorsque les différences d'échelle sont sujet de préoccupation, les chercheurs en comptabilité devraient faire intervenir un substitut d'échelle à titre de variable indépendante et formuler les inférences à partir des erreurs-types de White.

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There is a large and growing number of accounting research studies using levels-based designs (e.g., Bowen 1981; Daley 1984; Olsen 1985; Landsman 1986; Magliolo 1986; Harris and Ohlson 1987; Beaver, Eger, Ryan, and Wolfson 1989; Barth, Beaver, and Stinson 1991; Shevlin 1991; Kothari and Zimmerman 1995; Barth 1991, 1994; Barth, Beaver, and Landsman 1992, 1996; among others).[1] Two econometric issues in such studies are cross-sectional scale differences among sample firms that can result in biased coefficient estimates and heteroscedastic regression errors that can cause biased standard error estimates and estimation inefficiency. Scale differences arise because large (small) firms have large (small) values of many variables. These differences can result in heteroscedastic regression error variances and, if the magnitude differences are unrelated to the research question, scale-related coefficient bias. We focus on scale not because it is the only cross-sectional estimation issue, but because it is a pervasive and potentially important one.

We seek to provide evidence on the extent of scale-related econometric problems in accounting research contexts and the effectiveness of available remedies. We consider three primary available remedies for scale-related problems: deflating regression variables by a scale proxy, including a scale proxy as an independent variable, and using White (1980) heteroscedasticity-consistent standard error estimates. The econometrics literature provides theoretical guidance on the scale-related problems we investigate, but the guidance often is based on theoretical distributions and asymptotic properties. We use simulations based on COMPUSTAT firms and accounting data to mimic samples, empirical distributions of accounting variables, and estimation equation specifications typical in empirical financial accounting research. Although our analytical development is general, our selection of accounting and scale variables is motivated by regressions of, for example, market value of equity on net income or book value of equity. Our objective is to aid accounting researchers in developing effective research designs and in interpreting findings of accounting studies that use levels-based designs.[2]

As part of our analysis, we (1) develop expressions for coefficient bias that permit us to identify factors that cause bias, (2) develop expressions for heteroscedasticity-related standard error bias based on moments of the scale factor's distribution, permitting us to estimate the bias from observed variables, and (3) investigate the small sample properties of White (1980) standard error estimates and test for heteroscedasticity. To illustrate our simulation findings, we use three recent accounting studies, Kothari and Zimmerman (1995), Barth (1994), and Sougiannis (1994).[3]

Regarding coefficient bias, our analysis reveals that the bias magnitude increases with increases in the coefficients of variation of the independent variable and scale, which are large for variables typically encountered in accounting research. However, because the bias also depends on the "true regression" intercept which is unknown, one cannot estimate the magnitude of the bias. Evaluating our bias formulas using accounting variables indicates that proxies such as total assets, sales, book value of equity, net income, number of shares outstanding, and share price that are commonly used to deflate accounting regression variables typically mitigate coefficient bias only by a small amount, if at all.

Unfortunately, tractability limits our analytical development to regression models with only one independent variable and to cases when the scale factor and true independent variable are uncorrelated. Simulations permit us to remove these restrictions. Our simulations reveal that, surprisingly, simulated proxies 95 percent correlated with the true scale factors can *worsen* bias if used as deflators.[4] Yet they, and less highly correlated proxies, are quite effective at mitigating bias if included as independent variables. We find this holds regardless of whether the true independent variable and scale factor are uncorrelated.

Because most extant accounting research studies have more than one independent variable, we extend our simulations to regressions with two independent variables. The findings are similar to those from the univariate case, but also indicate that the coefficient on the variable more (less) highly correlated with scale is more (less) biased. This evidence suggests that coefficients on, for example, components of net income or particular assets or liabilities such as securities gains and losses or the fair value of investment securities, in estimation equations that also include a variable more highly correlated with scale, such as net income or total assets, likely are unaffected by scale.

Regarding heteroscedasticity, our simulations indicate that in specifications using undeflated variables, true standard errors can be five times as large as ordinary least squares (OLS) estimated standard errors, indicating severe standard error bias. However, White (1980) standard errors are close to the true ones, regardless of whether the errors are heteroscedastic. This finding is important because Chesher and Jewitt (1987) show that White standard errors can be biased in small samples, but there is little evidence on their finite sample accuracy. Moreover, the finding suggests that reporting White standard errors estimated from undeflated regressions is appropriate regardless of whether the regression error variances are heteroscedastic. *A priori*, it is not obvious that White standard errors are accurate in finite samples with homoscedastic errors because their calculation depends on estimated residuals rather than the identity matrix. We also find that White's test is effective at identifying heteroscedasticity in the samples we investigate. Contrary to one of its intended purposes, we find that deflation can result in efficiency losses–deflating by number of shares results in *losses* of up to 300 percent–even when the deflator is 95 percent correlated with the true scale factor.

In summary, our findings suggest that including a scale proxy as an independent variable and reporting inferences based on White standard errors is more effective than deflation as a remedy for econometric problems related to scale differences across firms. Also, our findings indicate that deflation has unpredictable effects on coefficient bias, heteroscedasticity, and estimation efficiency.

The remainder of the paper is organized as follows. The next two sections focus on scale-related coefficient bias, potentially in conjunction with heteroscedasticity. Section two explores the extent of scale-related coefficient bias and the third section presents empirical and simulation evidence. Section four explores heteroscedasticity in isolation. The final section summarizes and concludes.

Sources and extent of scale-related coefficient bias

Intuition and example

This section presents the model we use in developing an expression for scale-related coefficient bias to identify its sources. The model shows that one can view scale as an omitted regression variable and that deflation and including scale as an independent regression variable are two remedies for scale-related coefficient bias. We also develop an expression for the ratio of the bias in deflated and undeflated coefficient estimates when using a scale proxy as a deflator. Tractability limits our analytical development to simple regressions and cases in which the true scale factor, S, and true variable of research interest, X, are independent. In section three, we use simulations to explore coefficient bias and the effectiveness of the two remedies at mitigating it in simple and multiple regression contexts and in situations where S and X are correlated.

To develop intuition for our model of scale effects, assume a researcher seeks to study the relation between firms' equity market values and earnings. The research hypothesis is that market values depend on firms' earning power and the relation is linear. Of course, if the amount originally invested in a firm is large, other things equal, market value of equity also will be large. We refer to a variable such as the amount invested as "scale" and the market value of equity and net income as "observed variables." Scale affects the values of observed variables, but variation in equity market value attributable to variation in scale is not of research interest. An alternative way to view the researcher's hypothesis is that it relates to market value of equity and net income *after controlling for scale differences*. We refer to variables of research interest as "true variables" and coefficients in the regression of true dependent on true independent variables as "true coefficients." The researcher's challenge is to purge the scale factor's effect from the observed variables without purging the effect of the true independent variable, and theory often aids the researcher in this task. Unfortunately, however, scale often is not observed. In this example, book value of equity is a natural candidate scale proxy, but book value of equity reflects accounting earnings since the original investment and amounts invested at different times at different price levels. Thus, it does not equal the amount invested, that is, the scale factor.

Christie (1987) suggests depreciation expense as an accounting variable that differs with firm scale but has no economic relation to firm equity value. As Christie conjectures, untabulated findings from estimating a regression of market value of equity on depreciation using all COMPUSTAT firms with non-missing data for 1990 (nobs = 1,574) indicate that market value of equity is related significantly to depreciation expense (White $t = 7.27$). However, deflating market value of equity and depreciation by sales to remove scale differences results in a coefficient on depreciation indistinguishable from zero (White $t = -1.16$), and the R^2 drops from 0.40 to 0.01, suggesting that the observed relation between the undeflated variables is attributable to scale differences.[5]

Factors determining coefficient bias

To identify factors that determine coefficient bias, we assume that X_i and Y_i for firm i are variables of research interest and that scale has a multiplicative effect on X and Y. Thus, the researcher observes S_iX_i and S_iY_i. In terms of the example above, X and Y are unobservable earnings and market value of equity after controlling for scale, and SX and SY are observed net income and market value of equity.

Let the relation between X and Y be given by

$$Y_i = a + b_{Y.X}\, X_i + e_i\,, \tag{1}$$

where the error, e, is assumed homoscedastic. The researcher wants to test hypotheses about $b_{Y.X}$, for example, the earnings coefficient, either that it differs from zero or equals a specific amount. To obtain the relation between the observed variables, SX and SY, multiply equation 1 by S_i,

$$S_iY_i = a\, S_i + b_{Y.X}\, X_iS_i + e_iS_i\,. \tag{2}$$

In the regression of observed SY on only observed SX, that is,

$$S_iY_i = a' + b_{SY.SX}\, S_iX_i + \varepsilon_i\,, \tag{3}$$

$b_{SY.SX}$ does not necessarily equal $b_{Y.X}$ because equation 3 omits S_i and has an intercept, and equation 2 does not.[6] Also note that even if one includes S_i in equation 3 and suppresses the intercept, the error term, $\epsilon_i = S_ie_i$, is heteroscedastic because e is assumed to be homoscedastic.[7] Thus, scale-related coefficient bias and heteroscedasticity easily can coexist, and often do.

If one assumes the scale factor, S, is independent of X and Y, it can be shown that the expectation of the estimated coefficient on SX in equation 3, $E(b_{SY.SX})$, is

$$E(b_{SY.SX}) = b_{Y.X} + \frac{a \cdot \overline{X}/\mathrm{Var}(X)}{1 + \dfrac{\overline{S}^2}{\mathrm{Var}(S)} + \dfrac{\overline{X}^2}{\mathrm{Var}(X)}}. \tag{4}$$

Recall that X and Y are unobservable. Although in accounting contexts S might be correlated with X and Y, assuming it is not permits us to identify major factors that cause coefficient bias. We relax the independence assumption in simulations reported in section three.

The coefficient bias, $E(b_{SY.SX})$ minus $b_{Y.X}$, is the second term on the right-hand side of equation 4. Because the bias term's denominator always is positive, its sign depends on $\overline{X}$ and a, which are not observable. If both are positive, the bias is positive and $E(b_{SY.SX})$ exceeds $b_{Y.X}$. This could be true in accounting contexts because (1) most accounting variables such as net income and book value of equity have positive means, suggesting that $\overline{X}$ could be positive, and (2) negative intercepts are observed rarely in accounting relations such as between market value of equity and net income or book value of equity. Moreover, if the research null hypothesis is that X and Y are unrelated and the null is true, and $\overline{Y}$ is positive, the bias will be positive because the intercept is positive for any dependent variable with positive mean, that is, if $Y = a + 0\,X$, then $\overline{Y} > 0$ implies $a > 0$. Note that it is possible for $E(b_{SY.SX})$ to be positive with $b_{Y.X}$ negative, and vice versa. Note also that because X is unobservable, it is not possible to eliminate the bias simply by mean-differencing the independent variable so that $\overline{X}$ equals zero.

Equation 4 reveals that in a simple regression with X independent of S, only S's coefficient of variation affects the bias. Because the reciprocal of S's coefficient of variation appears in the bias term's denominator, the higher it is, the higher is the bias. Thus, if the coefficient of variation is small relative to that of X, then bias is small. However, because the denominator in equation 4 varies less than S's coefficient of variation, bias does not increase proportionally with the coefficient of variation.[8] Note also that the bias depends on a, the true intercept, which typically is unknown.

Techniques to mitigate the bias

If coefficient bias exists, the researcher must attempt to mitigate it. In the special case when the scale factor, S_i, is known, the researcher can divide the observed variables by the scale factor to obtain Y and X and estimate equation 1, because it fulfills all normal regression assumptions; deflation simultaneously cures coefficient bias and heteroscedasticity. Note that only the dependent and independent variables are deflated, not the intercept. Equation 2 shows that an alternative is to regress SY on S and SX, suppress the intercept, and use a procedure to correct for heteroscedasticity.[9] Deflation unambiguously is the better remedy if the true scale factor is known. However, when true scale is unknown and proxies must be used, it is an empirical question which of the two remedies is more effective at mitigating coefficient bias and heteroscedasticity. We provide evidence on this in sections three and four.

To determine whether coefficient bias is mitigated by deflating by a scale proxy, S', we note that deflation yields a different scale factor, $G_i = S_i / S_i'$, for each firm. Substituting G for S in equation 4, the ratio of the bias in the deflated coefficient estimate, $E(b_{GY.GX})$ minus $b_{Y.X}$, to the bias in the undeflated coefficient estimate, $E(b_{SY.SX})$ minus $b_{Y.X}$, is

$$\text{Bias Ratio} = \frac{1 + \dfrac{\overline{S}^2}{\text{Var}(S)} + \dfrac{\overline{X}^2}{\text{Var}(X)}}{1 + \dfrac{\overline{G}^2}{\text{Var}(G)} + \dfrac{\overline{X}^2}{\text{Var}(X)}}. \tag{5}$$

A ratio less than one indicates that deflation has reduced coefficient bias. If S' is a "good" deflator then the bias ratio is close to zero. This is true if $\overline{G}^2/Var(G)$ >> $\overline{S}^2/Var(S)$.

The magnitude of $\overline{G}^2/Var(G)$ depends on the joint distribution of S and S'. However it is impossible, in general, to derive the distribution of a ratio of two variables, S and S', in terms of the parameters of their joint distribution. Taylor approximations do not yield values for $\overline{G}^2/Var(G)$ close enough to the true ones to enable us to rely on the approximations for making statements about the ratio's characteristics. Thus, we only note that the magnitude of $\overline{G}^2/Var(G)$ depends not only on the correlation between S and S' but also on the intercept in the regression of S on S', and possibly on higher order moments of S and S' than two. In section three, we evaluate equation 5 using accounting variables and use simulations to explore effects of deflation using a proxy for scale.

Empirical and simulation evidence on scale-related coefficient bias

This section provides evidence on coefficient bias and the available remedies' effectiveness in accounting contexts. We base our descriptive statistics and simulation findings on four samples derived from 1990 COMPUSTAT data after deleting firms with net income or book value of equity less than 0.01: (1) all firms (1,773 observations), (2) firms with total assets in excess of $1 billion (738 observations), (3) the 500 largest firms in terms of total assets, and (4) 100 firms randomly selected from the 500-firm sample. These are intended to represent samples typically encountered in empirical accounting research. Because our findings generally are similar across the four samples, we report only those for the 500-firm sample and note differences across samples when appropriate. As described in Appendix 1, we also use estimation equations similar to those in Kothari and Zimmerman (1995), Barth (1994), and Sougiannis (1994) to illustrate our findings.

TABLE 1
Correlations between variables used as scale factors, deflators, and independent variables and $\bar{S}^2/Var(S)$ and $\bar{G}^2/Var(G)$ for different combinations of assumed true scale factors, *S*, and deflators, *S'*.

Panel A: Pearson (*Spearman*) correlations in the upper (*lower*) triangle.

	TA	*SALES*	*BVE*	*NI*	*NUMSHR*	*PRICE*
TA	—	0.593 (0.0001)	0.532 (0.0001)	0.387 (0.0001)	0.378 (0.0001)	-0.009 (0.8410)
SALES	*0.577* *(0.0001)*	—	0.843 (0.0001)	0.750 (0.0001)	0.655 (0.0001)	-0.001 (0.9984)
BVE	*0.659* *(0.0001)*	*0.840* *(0.0001)*	—	0.847 (0.0001)	0.668 (0.0001)	0.044 (0.3374)
NI	*0.528* *(0.0001)*	*0.777* *(0.0001)*	*0.845* *(0.0001)*	—	0.758 (0.0001)	0.027 (0.5515)
NUMSHR	*0.478* *(0.0001)*	*0.756* *(0.0001)*	*0.823* *(0.0001)*	*0.798* *(0.0001)*	—	-0.030 (0.5146)
PRICE	*0.122* *(0.0078)*	*0.431* *(0.0001)*	*0.417* *(0.0001)*	*0.524* *(0.0001)*	*0.198* *(0.0001)*	—

Panel B: $\bar{S}^2/Var(S)$ and $\bar{G}^2/Var(G)$

True scale		$\bar{G}^2/Var(G)$ by deflator					
factor (S)	$\bar{S}^2/Var(S)$	*TA*	*SALES*	*BVE*	*NI*	*NUMSHR*	*PRICE*
TA	0.357	—	**0.863**	**0.692**	0.152	0.002	0.137
SALES	0.404	**1.203**	—	**1.098**	0.056	0.001	0.377
BVE	0.473	**2.362**	**2.231**	—	0.120	0.002	**0.684**
NI	0.420	**1.168**	**2.066**	**1.092**	—	0.001	**0.770**
NUMSHR	0.604	**0.818**	**1.037**	**1.091**	0.045	—	0.260
PRICE	0.026	**0.055**	**0.028**	**0.207**	0.018	0.002	—

Based on sample of 500 largest (in terms of total assets) COMPUSTAT firms for 1990.The p-values for testing the statistical significance of the correlations are in parentheses.
TA = total assets; *SALES* = sales; *BVE* = book value of equity; *NI* = net income; *NUMSHR* = number of shares outstanding; *PRICE* = share price. $G = S/S'$. If $\bar{G}^2/Var(G)$ is larger than $\bar{S}^2/Var(S)$, deflation by the scale proxy, *S'*, reduces the coefficient bias caused by scale differences when the true scale factor is *S*. The cases when this is true are in boldface type.

Descriptive statistics on commonly used accounting scale proxies

Table 1, panel A presents correlations among the variables we use in our analyses. Spearman correlations indicate that the variables are significantly positively correlated, although the correlations range from 0.122 to 0.845. Pearson correlations are similar except that they indicate that share price, *PRICE*, is not significantly correlated with the other variables.

Because the relation among S, S', and $\bar{G}^2/Var(G)$ in equation 5 is complex, we provide empirical and simulation evidence about the effect of deflating by scale proxies, S'. Table 1, panel B presents results of evaluating equation 5 for several accounting variables assuming they are S and S'. We assume that total assets, *TA*, sales, *SALES*, book value of equity, *BVE*, net income, *NI*, number of shares outstanding, *NUMSHR*, or share price, *PRICE*, is the true unobservable scale factor, S, but the researcher deflates by S' which is one of the other variables. For completeness and because the true scale factor is unknown, we present $\bar{S}^2/Var(S)$ and $\bar{G}^2/Var(G)$ for all resulting combinations of S and S'. Comparisons among alternative deflators in panel B are relevant only along the table's rows because in any particular situation the true scale factor is a given and enters the calculation of $\bar{G}^2/Var(G)$. Comparisons across columns reveal only whether particular deflators mitigate bias across alternative true scale factors. This potentially is informative because the true scale factor is unknown.

Panel B reveals that using *TA*, *SALES*, or *BVE* as a deflator reduces coefficient bias, that is, $\bar{G}^2/Var(G) > \bar{S}^2/Var(S)$. For example, when *TA* is the deflator and *BVE* is the true scale factor, $\bar{G}^2/Var(G)$ equals 2.362 and $\bar{S}^2/Var(S)$ equals 0.473. Using *NI*, *NUMSHR*, or *PRICE* as deflators generally does not mitigate coefficient bias. The exceptions are when *BVE* or *NI* is the true scale factor and *PRICE* is the deflator. However, although deflating by *TA*, *SALES*, or *BVE* reduces coefficient bias for the true scale factors we consider, the magnitude of improvement is small. Equation 5 shows that the bias ratio depends on the coefficients of variation of X, S, and G. Assuming X's coefficient of variation is the same as S's, the smallest bias ratio is 51 percent [(1 + 0.473 + 0.473) / (1 + 2.362 + 0.473)] when *BVE* is the true scale factor and *TA* is the deflator. Thus bias is reduced, at best, by approximately one-half.[10] The smallest bias ratio for the other assumed true scale factors ranges from 53 to 85 percent, indicating bias is reduced by only 15 to 47 percent. In the worst case, bias is *increased* by 34 percent.

Coefficient bias diagnostic

The model in section two indicates scale-related coefficient bias results from omitting scale as an independent regression variable. Example 1 in Appendix 2 also illustrates how scale differences can cause coefficient bias. It shows that because bias results from omitting scale as an independent variable, effectively there are different intercepts for observations that differ in scale. Consequently, a diagnostic for identifying possible coefficient bias is to partition the sample into groups based on an assumed scale factor and test whether the groups' intercepts differ. If they do, then biased slope coefficients likely will result from estimating a regression with a common intercept for the full sample. This diagnostic is similar in spirit to the Goldfeld-Quandt (1965) heteroscedasticity test and requires assuming only that the relation between the observed variable and scale factor is monotonic for a given value of the true variable.[11] Of course, the diagnostic is limited to the extent the partitioning scheme is noisy because the true scale factor is unknown. Elimination of the middle 10 or 20 percent of the observations can mitigate noise effects.

To illustrate the diagnostic, we partition the depreciation sample above into two groups based on sales. Untabulated findings reveal that the two intercepts differ significantly (146.97 and 1,190.24, White $t = 3.75$), indicating that depreciation's coefficient in the initial estimation could be biased because of scale differences. We confirm this by estimating:

$$MVE = a + b\ DEPR + c\ SALESDEPR + e$$

where *MVE* is market value of equity, *DEPR* is depreciation expense, and *SALESDEPR* equals depreciation times an indicator variable that equals one for firms with *SALES* greater than the sample median and zero otherwise. Untabulated findings reveal a significantly positive coefficient on *SALESDEPR*–*c*'s estimate is 173.34 with a White *t*-statistic of 5.65.

To use Barth (1994) for illustrating the diagnostic, we partition Barth's 1989 sample based on a scale proxy, book value of equity for the investment securities regressions, and net income for the securities gains and losses regressions. Regression summary statistics reported in Table 2 indicate that scale potentially is an omitted variable for both regressions–the intercepts for the two size groups statistically differ at less than the 0.02 level using White standard errors. We confirm this by estimating equations using a scale proxy as a deflator and as an independent variable.

TABLE 2

Illustrations of scale-related coefficient bias diagnostic. Estimates of specifications related to Barth (1994) regressions of equity market value on the difference between fair and book values of investment securities, *FVBV*, and unrealized securities gains and losses, *URSGL*, partitioned on median of a scale proxy (book value of equity for investment securities regression and net income for securities gains and losses regression). Based on Barth (1994) sample of all COMPUSTAT banks for 1989 with available data and non-negative book value of equity or net income. White (1980) standard errors are in parentheses.

	Coefficients (standard errors)			
	Investment securities		Securities gains and losses	
	Intercept	*FVBV*	Intercept	*URSGL*
Big	1,957.70 (253.55)	12.37 (5.34)	1,172.08 (468.28)	17.60 (6.85)
Small	284.64 (29.02)	4.12 (5.72)	166.79 (32.66)	17.96 (2.27)
adj R^2	0.59		0.55	
nobs	139		122	

Evidence on using a scale proxy to mitigate coefficient bias
This section presents evidence on whether deflation by a scale proxy or its inclusion as an independent variable results in less coefficient bias. Our findings suggest that common deflators might not mitigate coefficient bias because they are insufficiently highly correlated with true scale factors. We use simulations to investigate coefficient estimates and their true and estimated standard errors when the deflator, or scale proxy, and the true scale factor are highly correlated.

Table 3 presents findings for when the true scale factor, *S*, is book value of equity, *BVE*, and the scale proxy, *S'*, is simulated to be 95 percent correlated with *S*. The analytical development in section two assumes the true scale factor, *S*, and independent variable, *X*, are independent, and relates only to regressions with one independent variable. Table 3 reports simulation results when *S* and *X* are independent and when they are 0.50 correlated. We discuss, but do not tabulate, findings for other correlations. Figure 1 graphs coefficient bias when using a scale proxy as a deflator or an independent variable for correlations between *S* and *S'* ranging from 0.50 to 0.99, when *S* and *X* are uncorrelated, and the standard deviation of *S* - *S'* is proportional to *S*. In a following section, we consider the case of multiple independent variables.

TABLE 3
Means from 250 simulation iterations of estimating seven models of true regression, $Y = a + b X + u$, where *X* is a random variable, $Y = 1{,}500 + 7X + e$, and *e* is homoscedastic and normally distributed with mean zero. In some models, only scaled variables, *SX* and *SY*, are observed, and not *X* and *Y*. *S* is book value of equity. Some models assume *S* is observable, others assume it is not. *S'* is a proxy for *S* where *S'* is simulated to have 0.95 correlation.

			Standard error of *b*			
Model	Assumption about $\sigma(S - S')$	Estimate of *b* minus true value of 7	OLS	White	Estimate of true	White / estimate of true
		Corr (*S*,*X*) = 0				
1.		-0.011	0.317	0.310	0.317	0.976
2.		5.469	0.213	0.746	0.809	0.922
3.	independent	6.801	0.168	1.132	3.919	0.289
	proportional	2.914	0.286	0.636	1.613	0.394
4.	independent	1.622	0.301	0.889	1.062	0.837
	proportional	1.536	0.302	1.017	1.285	0.792
5.		-0.048	0.303	0.997	1.277	0.781
6.	independent	0.050	0.315	1.624	4.141	0.392
	proportional	0.010	0.310	0.475	0.861	0.552
7.	independent	4.609	0.231	1.315	3.464	0.380
	proportional	2.747	0.290	0.599	1.568	0.382

		Corr $(S,X) = 0.5$				
1.		-0.010	0.309	0.308	0.315	0.977
2.		2.951	0.090	0.419	0.486	0.863
3.	independent	7.751	0.184	1.232	3.778	0.326
	proportional	2.890	0.284	0.628	1.461	0.430
4.	independent	1.385	0.178	0.564	0.785	0.719
	proportional	1.344	0.180	0.640	0.884	0.724
5.		0.045	0.209	0.711	1.068	0.666
6.	independent	0.050	0.359	1.843	4.740	0.389
	proportional	0.003	0.315	0.485	0.824	0.588
7.	independent	5.077	0.244	1.366	3.654	0.374
	proportional	2.810	0.281	0.579	1.348	0.430

Model:

1. $Y = a_1 + b_1 X + e_1$
2. $SY = a_2 + b_2 SX + e_2$
3. $GY = a_3 + b_3 GX + e_3$
4. $SY = a_4 + b_4 SX + c_4 S' + e_4$
5. $SY = a_5 + b_5 SX + c_5 S + e_5$
6. $GY = a_6 1/S' + b_6 GX + c_6 G + e_6$
7. $GY = a_7 1/S' + b_7 GX + c_7 + e_7$

Based on sample of 500 largest (in terms of total assets) COMPUSTAT firms for 1990. OLS and White refer to the estimated OLS or White (1980) standard errors of b, the coefficient on X, SX, or GX. The estimate of the true standard error is the standard deviation of the coefficient estimated over the 250 iterations. $G = S/S'$.

We simulate $X = a\,(BVE - 3{,}181) + b\,Z + 200$ where $a = 0$ (0.01081) and $b = 100$ (86.59) for the case of X and S, that is, BVE, independent (0.5 correlated). Z is an independent standard normal random variable, and 3,181 is the sample mean of BVE. Thus, X has mean and variance equal to 200 and 100^2. $Y_i = 1{,}500 + 7\,X_i + e_i$ where the e_i are homoscedastic, normally, and independently distributed with mean zero and variance equal to 700^2, resulting in a regression R^2 of 0.50. The coefficient value of seven is arbitrary, and the intercept of 1,500 is chosen to induce large coefficient bias. Our findings are insensitive to these amounts because, as equation 5 reveals, neither affects the bias ratio. Our scale proxy, S', equals $S + v$ where v is normally distributed with mean zero. We present findings under two assumptions about the standard deviation of v, (1) that it equals 1,521 and (2) that it equals (1,521 BVE)/5,616, that is, it is proportional to the scale factor, BVE. Both assumptions insure 95 percent correlation.[12] We present both because in some situations one may be more descriptive than the other, although because the true scale factor is unobservable, the researcher may not know which. As more fully discussed below, our inferences are similar under both assumptions. G is as previously defined.

We estimate seven regression models.

1. $Y = a_1 + b_1 X + e_1$
2. $SY = a_2 + b_2 SX + e_1{*}S + a_1(S - \bar{S})$
3. $GY = a_3 + b_3 GX + e_1{*}G + a_1(G - \bar{G})$
4. $SY = a_4 + b_4 SX + c_4 S' + e_1{*}S + a_1(S - \bar{S}) + c_4(S' - \bar{S}')$
5. $SY = a_5 + b_5 SX + c_5 S + e_1{*}S$
6. $GY = a_6\, 1/S' + b_6 GX + c_6 G + e_1{*}G$
7. $GY = a_7\, 1/S' + b_7 GX + c_7 + (e_1{*}S + a_1(S - \bar{S}) + c_4(S' - \bar{S}'))/S'$

Model 1 is the benchmark because it uses true variables, and thus *X*'s coefficient estimate will be unbiased and there is no heteroscedasticity. Model 2 uses assumed observed variables that are affected by scale, *SY* and *SX*. As in equation 3, estimating model 2 results in coefficient bias and heteroscedasticity. Model 3 is the regression of observed *SY* on *SX*, both deflated by the scale proxy, *S'*, and an intercept. If *S'* equaled *S* then model 3 would be the same as model 1. Thus, whether model 3 has coefficient bias and/or heteroscedasticity depends on the ability of a scale proxy to mitigate these scale-related problems. Model 4 (model 5) includes the scale proxy (true scale factor) as an independent variable rather than as a deflator. Again, whether model 4 has coefficient bias depends on the effectiveness of the scale proxy at mitigating it; as equation 2 indicates, model 5 should have no coefficient bias.[13]

Because including scale as an independent variable addresses only coefficient bias, models 4 and 5 have heteroscedastic errors. Section four indicates that deflating all regression variables including the intercept is a remedy for heteroscedasticity. Thus, we estimate models 6 and 7, which are deflated versions of models 5 and 4, to investigate deflation by a scale proxy as a remedy for heteroscedasticity in these models. In section four, we explore deflation as a remedy for heteroscedasticity when heteroscedasticity is known to be the only econometric concern.

Table 3 presents means of 250-simulation iterations when the standard deviation of *S* - *S'* is independent of *S* and proportional to *S*. Consider first the case when *S* and *X* are independent. As expected, differences in coefficient estimates between models 1 and 2 are statistically significant indicating coefficient bias. Heteroscedasticity in model 2 also causes estimated coefficient standard error to be less than the true one, 0.213 versus 0.809, where the estimated true standard error is the standard deviation of estimates of *b* over the 250 iterations. Surprisingly, coefficient bias in model 3 is worse, and the true standard error is higher than in model 2. This suggests that deflating by a proxy highly correlated with the true scale factor *worsens* coefficient bias and *decreases* efficiency.[14] Strikingly, in model 4, where the scale proxy is an independent variable rather than a deflator, coefficient bias is much smaller, 1.622 versus 6.801. Also as expected, there is no coefficient bias in model 5, where the true scale factor is included as an independent variable, and coefficient estimates in models 4 and 5 have higher standard errors than those in model 1, presumably attributable to heteroscedasticity. These findings suggest including a scale proxy as an independent variable is more effective than deflation as a remedy for scale-related coefficient bias.

In model 6, the deflated version of model 5, deflation by a scale proxy is intended to mitigate heteroscedasticity, the only econometric problem in model 5. The findings indicate that estimation efficiency as measured by the standard deviation of the coefficient estimates *decreases* for model 6 relative to model 5, 4.141 versus 1.277.[15] In model 7, the deflated version of model 4, coefficient bias and estimation efficiency are approximately three times *worse* than in model 4. These findings are consistent not only with those above that deflation can increase coefficient bias but also with those in section 4 that indicate using deflation as a remedy for heteroscedasticity when there is no coefficient bias can decrease estimation efficiency.

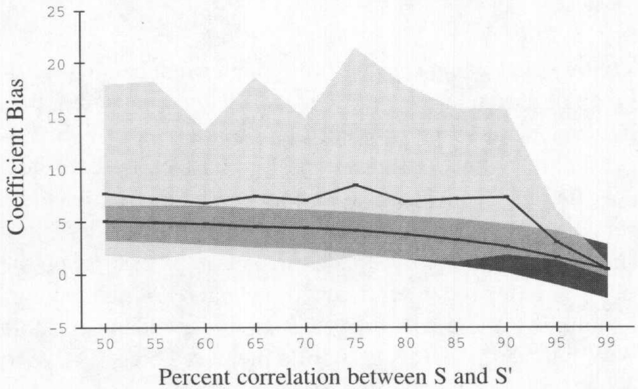
Findings from assuming the standard deviation of S - S' is proportional to S generally are similar to the independence case. The most notable differences are (1) deflation in model 3 mitigates coefficient bias by almost one-half. However, including the scale proxy as an independent variable in model 4 is more effective than deflation in model 3 at mitigating coefficient bias and is more efficient. (2) Deflation in model 6 increases estimation efficiency somewhat relative to model 5, consistent with heteroscedasticity being mitigated. (3) Deflation in model 7 results in less coefficient bias and greater estimation efficiency than in the independence case. However, consistent with the independence case findings, bias and efficiency are worse in model 7 than in model 4 where there is heteroscedasticity and no deflation.

Assuming S and X are 50 percent correlated results in inferences similar to assuming S and X are independent, except that the magnitude of the scale bias, that is, the bias in model 2, is less, 2.951 versus 5.469. Untabulated findings indicate that these inferences also are valid for other correlations between S and X; as correlation increases, bias decreases, but all inferences regarding using a scale proxy to mitigate bias are the same as for the two reported correlations.

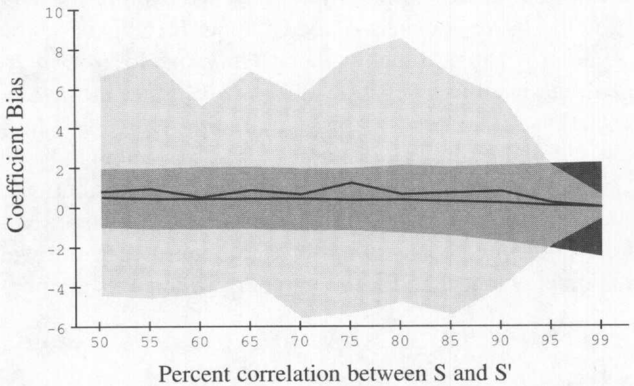
The Table 3 findings are based on assuming the scale proxy, S', is 95 percent correlated with the true scale factor, S. Figure 1, panel A graphs coefficient bias and estimated true coefficient standard error when using the scale proxy as a deflator or an independent variable, models 3 and 4, for simulations similar to those in Table 3 except that the correlations between S and S' range from 0.50 to 0.99. Figure 1 graphs only the case when S and X are uncorrelated and the standard deviation of S - S' is proportional to S because differences in coefficient bias and estimation efficiency between using a scale proxy as a deflator and an independent variable are smaller in that case than when the standard deviation is independent of S.[16]

Figure 1 Graph of means and empirical 95 percent confidence intervals from 250 simulation iterations of coefficient bias in two specifications of regression, $SY = a + b\ SX + Su$, where X = is a random variable uncorrelated with S, $Y = a + 7X + e$, and e is homoscedastic and normally distributed with mean zero. S is book value of equity. The top line and lighter confidence interval relates to estimating the regression deflated by S', a proxy for S (Model 3: $GY = a_3 + b_3\ GX + e_3$) where the standard deviation of $S - S'$ is proportional to S. The lower line and darker confidence interval relates to estimating the regression with S' included as an independent variable (Model 4: $SY = a_4 + b_4\ SX + c_4\ S' + e_4$). $G = S/S'$.

Panel A: Intercept, a, equals 1,500



Panel B: Intercept, a, equals 150



Panel C: Intercept, *a*, equals 15

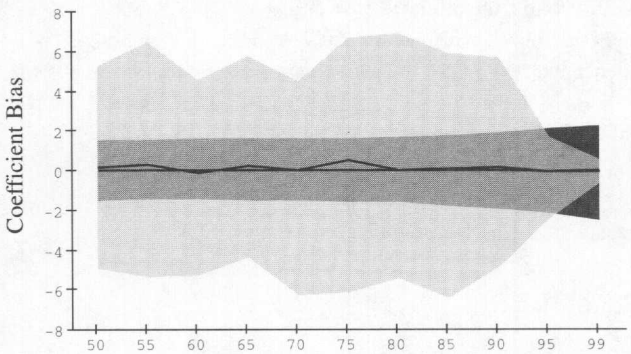


Figure 1, panel A shows that using a scale proxy as an independent variable is more effective than deflation at mitigating scale-related coefficient bias for all correlations. However, with 99 percent correlation, there is little difference between the specifications—in both cases the bias is small (0.408 and 0.326)—except that including the scale proxy as an independent variable is less efficient (standard errors equal 0.326 and 1.283 in the deflation and independent variable cases). Untabulated findings indicate this is not the case when the standard deviation of S - S' is independent of S, and S and S' are 99 percent correlated. In that case, including the scale proxy as an independent variable almost eliminates the bias but, as in Table 3, deflation increases bias relative to model 2.

The simulations reported in Table 3 and Figure 1, panel A are based on assuming the true intercept equals 1,500. Because we selected a large intercept to induce large coefficient bias for expositional purposes, we explore the sensitivity of this assumption in Figure 1, panels B and C. Panel B (C) is based on an assumed true intercept of 15 (150). Arguably, these likely are closer to intercepts observed in typical applications than is 1,500. For example, in the two Kothari and Zimmerman (1995) models in Appendix 1, the intercepts range from 18 to 105. As expected, the findings in Figure 1, panels B and C indicate that coefficient bias is lower than reported in panel A. However, consistent with including a scale proxy as an independent variable being a more effective remedy than deflation, under both intercept assumptions, the confidence interval for the deflated model is much wider than for the model in which S' is an independent variable.

Illustrations using recent studies

Kothari and Zimmerman (1995) deflate regression variables by number of shares, *NUMSHR*. Thus, we use Kothari and Zimmerman to investigate effects on the estimated net income coefficient, ERC, when, as an alternative to deflation, scale proxies are included as independent variables and whether deflating by *NUMSHR* increases coefficient bias as Table 1 suggests.

TABLE 4
Illustrations of scale effects, deflation, and using a scale proxy as an independent variable

Panel A: Kothari and Zimmerman (1995). Equity market value is dependent variable. Models 1–4 are undeflated, models 5 and 6 are deflated by number of shares outstanding.

Model	Intercept	*NI*	*NUMSHR*	*BVE*	adj R^2	White χ^2	White p
1	104.89	11.60			0.77	47.87	0.0000
	(29.62)	(0.45)					
2	24.85	10.02	5.79		0.78	45.59	0.0000
	(44.42)	(0.70)	(2.28)				
3	79.01	9.78		0.30	0.78	55.48	0.0000
	(28.46)	(0.73)		(0.10)			
4	11.69	8.65	5.15	0.26	0.79	56.41	0.0000
	(38.72)	(0.79)	(2.10)	(0.09)			
		EPS		*BVEPS*			
5	18.27	2.93			0.18	7.03	0.0297
	(1.90)	(0.96)					
6	18.47	3.39		-0.07	0.18	15.94	0.0070
	(1.87)	(1.43)		(0.09)			

Panel B: Barth (1994). Equity market value is dependent variable. Models 1–3 relate to investment securities (n = 139), models 4–6 relate to securities gains and losses (n = 122).

Model	Intercept	*FVBV*	*NI*	*BVE*	adj R^2	White χ^2	White p
1	1,066.58	16.62			0.20	2.40	0.3016
	(147.70)	(5.52)					
2	122.47	5.47		1.04	0.92	8.45	0.1333
	(40.03)	(2.22)		(0.05)			
		FVBV/BVE					
3	1.23	4.18			0.05	3.28	0.1945
	(0.04)	(1.70)					
		URSGL					
4	492.41	23.46			0.29	2.07	0.3559
	(198.20)	(4.88)					
5	32.85	0.67	9.94		0.73	9.57	0.0883
	(99.23)	(6.91)	(1.90)				
		URSGL/NI					
6	8.24	20.92			0.84	2.49	0.2879
	(4.83)	(0.66)					

Panel C: Sougiannis (1994). Net income (models 1 and 3) or net income scaled by total assets (models 2 and 4) is dependent variable. Sample for models 1 and 2 (models 3 and 4) are all COMPUSTAT firms with assets greater than $1 billion and available R&D data, n = 266, (available R&D data, n = 853).

Model	Intercept	*TA*	*R&D*	adj R^2	White χ^2	White p
1	158.24	0.04	1.65	0.88	19.32	0.0020
	(31.27)	(0.01)	(0.23)			
	1/TA	Intercept	*R&D/TA*			
2	-4.01	0.06	1.77	0.46	10.07	0.0030
	(11.96)	(0.01)	(0.20)			
	Intercept	*TA*	*R&D*			
3	35.41	0.04	1.66	0.89	16.15	0.0060
	(11.62)	(0.01)	(0.26)			
	1/TA	Intercept	*R&D/TA*			
4	-0.88	0.08	-0.11	0.34	19.23	0.0020
	(0.27)	(0.01)	(0.12)			

Kothari and Zimmerman (1995): regressions of equity market value on various combinations of undeflated net income, *NI*, book value of equity, *BVE*, and number of shares, *NUMSHR*, and similar specifications estimated using number of shares as the deflator. Based on sample of all COMPUSTAT firms with non-missing data and non-negative net income for 1989 (nobs = 1,906). *EPS* and *BVEPS* denote net income (or earnings) and book value of equity per share. 32 observations with values of *MVE, NI, PRICE*, or *EPS* more than five standard deviations from the mean are eliminated.

Barth (1994): regressions of equity market value on the difference between fair and book values of investment securities, *FVBV*, and unrealized securities gains and losses, *URSGL*, (1/4) undeflated, (2/5) deflated by the scale proxy, and (3/6) including the scale proxy as an independent variable. Based on Barth (1994) sample of all COMPUSTAT banks for 1989 with available data and non-negative book value of equity or net income.

Sougiannis (1994): regressions of net income, *NI*, on total assets, *TA*, and research and development expenditures, *R&D*, undeflated and deflated by total assets. Based on two samples of COMPUSTAT firms for 1989.

White (1980) standard errors are in parentheses.

Table 4, panel A reveals that when either *NUMSHR* or *BVE* or both are included as independent variables in models 2–4, their coefficients significantly differ from zero and the ERC estimate is smaller than that obtained when *NI* is the only independent variable (11.60 in model 1 versus 10.02, 9.78, and 8.65 in models 2, 3, and 4). However, the difference's economic magnitude is modest. Moreover, all estimates are reasonable when interpreted as a cost of capital and are substantially larger than Kothari and Zimmerman's ERC estimate in the returns specification, 0.45. Thus, Kothari and Zimmerman 's inference that the price model yields economically more sensible ERC estimates than the returns model largely are unaffected by scale.

Regarding effects of share deflation, Table 4, panel A also indicates that in model 5 when earnings per share, *EPS*, is the only independent variable, the ERC estimate is quite small, 2.93, indicating a cost of capital of 34 percent. When book value of equity per share, *BVEPS*, is included in model 6, the ERC estimate increases only by a small amount to 3.39. Moreover, *BVEPS*'s coefficient is negative and insignificantly different from zero. Because we consistently obtain reasonable coefficient estimates in undeflated specifications, even when a scale proxy is included as an independent variable, and unreasonable estimates in deflated specifications, we conclude that inferences from deflated regressions are suspect.[17]

We use Barth (1994) to demonstrate that inferences can differ if scale and deflation effects are not considered. Findings regarding investment securities in Table 4, panel B indicate that in model 1 when the difference between fair and book values of investment securities, *FVBV*, is the only independent variable, its coefficient is significantly positive (coef. est. = 16.62 and White t = 3.02). When a scale proxy, book value of equity, *BVE*, is included as an independent variable in model 2 or as a deflator in model 3, the coefficient estimates are much smaller than in model 1, 5.47 and 4.18 versus 16.62. However, whether scale is an independent variable or a deflator has little effect on the inferences–the coefficient estimates are similar and both statistically significantly differ from zero. This finding is consistent with Table 1 that indicates using book value of equity as a deflator can reduce coefficient bias.

However, findings regarding securities gains and losses in models 4–6 suggest different inferences. When the scale proxy is an independent variable in model 5, the coefficient estimate drops from 23.46 in model 4 to 0.67 and becomes statistically indistinguishable from zero (White t = 0.10).[18] When a scale proxy is the deflator in model 6, the coefficient estimate drops only to 20.92 and remains significantly different from zero. These findings are consistent with those in Table 3 and with omitting scale as an independent variable resulting in coefficient bias. Of course, because the true coefficient is unknown, it is not possible to determine definitively whether coefficients in models 4 and 6 are biased. However, Table 1 indicates that using net income as a deflator can increase coefficient bias. Moreover, this securities gains and losses specification is only one that Barth (1994) estimates when concluding that unrealized securities gains and losses add little explanatory power in explaining bank share prices. Other specifications in Barth (1994) that yield similar inferences include those using different deflators and using returns as the dependent variable.[19]

Sougiannis (1994) deflates regression variables by total assets when investigating research and development (R&D) expenses. Although Table 1 indicates that deflation by total assets reduces coefficient bias in our samples, findings in sections two and three suggest that any deflation can be problematic. Thus, we investigate the sensitivity to deflation of Sougiannis' findings with two samples of firms. For models 1 and 2 (models 3 and 4) we use all COMPUSTAT firms with assets greater than $1 billion and available R&D data, n = 266 (available R&D data, n = 853).

Table 4, panel C reveals that deflated estimation findings from model 2 are similar to undeflated findings from model 1, suggesting Sougiannis' R&D findings are unaffected by coefficient bias related to scale or by deflation. However, estimating models 3 and 4 illustrates potential problems with deflation. The coefficient on *R&D* in the undeflated regression, model 3, is similar to that in model 1, 1.66 versus 1.65, with similar standard errors. Even though Table 1 indicates using *TA* as a deflator reduces coefficient bias for the true scale factors we assume there, the coefficient on *R&D/TA* in the deflated regression in model 4 is negative, suggesting increased coefficient bias with deflation. This estimated coefficient differs insignificantly from zero and significantly from the coefficient in the undeflated regression in model 3. Without knowing the true coefficient, one cannot definitively determine whether the deflated coefficient is biased, but it is markedly different from those in all of the other specifications in panel C.[20]

TABLE 5
Estimated coefficient bias and true standard errors from simulations of seven models of true regression, $Y = a + b_1X1 + b_2X2 + u$, where $X1$ and $X2$ are random variables, $Y = 1{,}500 + 7X1 + 7X2 + e$, and e is homoscedastic and normally distributed with mean zero. In some models, only scaled variables, *SX1, SX2,* and *SY,* are observed and not *X1, X2,* and *Y*. *S* is book value of equity. Some models assume *S* is observable, others assume it is not. *S′* is a proxy for *S* where *S′* is simulated to have 0.95 correlation. Correlations between variables are in parentheses.

Panel A: $\sigma(S - S')$ independent of S

Model	(SX1, S; SX2, S)	b_1 bias	b_1 true sd	b_2 bias	b_2 true sd	b_1 bias	b_1 true sd	b_2 bias	b_2 true sd
		(S, X1) = (S, X2) = 0.0							
		(X1, X2) = 0.0				(X1, X2) = 0.5			
1.	na	-0.015	0.604	-0.002	0.618	-0.016	0.729	0.004	0.757
2.	equal	6.632	2.434	6.184	2.372	6.236	3.420	5.579	3.322
	0.8;0.5	10.919	2.324	4.316	3.527	12.675	2.898	-1.621	4.406
3.	equal	6.427	8.801	7.787	10.329	6.861	11.920	6.696	12.935
	0.8;0.5	13.215	11.210	4.909	17.673	14.529	9.274	0.529	17.808
4.	equal	2.942	2.212	2.586	2.252	2.320	2.640	1.790	2.769
	0.8;0.5	3.052	2.250	1.078	2.600	3.296	2.500	-0.543	2.991
5.	na	0.164	2.413	-0.105	2.503	0.220	2.595	-0.189	2.778
6.	na	-0.412	6.989	0.096	7.375	-0.416	8.296	0.078	8.701
7.	equal	5.473	8.442	6.045	8.577	5.114	10.691	5.477	11.368
	0.8;0.5	8.875	9.120	3.293	11.657	9.883	9.431	-0.850	13.847
		(S, X1) = (S, X2) = 0.5							
		(X1, X2) = implied = 0.28							
1.	na	-0.021	0.637	-0.003	0.650				
2.	equal	3.212	2.346	2.884	2.333				
3.	equal	7.048	9.971	8.505	11.519				
4.	equal	1.728	2.196	1.373	2.137				
5.	na	0.224	2.247	-0.188	2.224				
6.	na	-0.497	7.932	0.201	8.433				
7.	equal	5.787	9.382	6.483	9.483				

Panel B: $\sigma(S - S')$ proportional to S.

Model ($SX1$, S; $SX2$, S)		b_1 bias	b_1 true sd	b_2 bias	b_2 true sd	b_1 bias	b_1 true sd	b_2 bias	b_2 true sd
		(S, $X1$) = (S, $X2$) = 0.0							
		($X1$, $X2$) = 0.0				($X1$, $X2$) = 0.5			
1.	na	-0.015	0.604	-0.002	0.618	-0.016	0.729	0.004	0.757
2.	equal	6.632	2.434	6.184	2.372	6.236	3.420	5.579	3.322
	0.8;0.5	10.919	2.324	4.316	3.527	12.675	2.898	-1.621	4.406
3.	equal	4.132	2.110	4.146	2.317	3.357	2.462	3.486	2.976
	0.8;0.5	5.162	3.089	2.479	4.083	5.727	4.659	-0.089	5.019
4.	equal	2.696	2.498	2.399	2.528	2.070	2.895	1.626	2.968
	0.8;0.5	2.720	2.615	1.012	2.783	2.907	2.877	-0.459	3.199
5.	na	0.164	2.413	-0.105	2.503	0.220	2.595	-0.189	2.778
6.	na	-0.034	1.650	0.102	1.357	-0.053	1.874	0.127	1.690
7.	equal	6.236	1.697	6.343	1.913	5.633	2.190	6.021	2.735
	0.8;0.5	10.357	1.911	4.672	3.320	12.040	3.887	-0.722	4.483
		(S, $X1$) = (S, $X2$) = 0.5							
		($X1$, $X2$) = implied							
1.	na	-0.021	0.637	-0.003	0.650				
2.	equal	3.212	2.346	2.884	2.333				
3.	equal	3.734	2.251	3.801	2.609				
4.	equal	1.574	2.309	1.279	2.237				
5.	na	0.224	2.247	-0.188	2.224				
6.	na	-0.067	1.751	0.110	1.373				
7.	equal	5.775	1.867	5.937	2.152				

Model:

1.	Y	=	a_1	+	$b_1 X$			+	e_1
2.	SY	=	a_2	+	$b_2 SX$			+	e_2
3.	GY	=	a_3	+	$b_3 GX$			+	e_3
4.	SY	=	a_4	+	$b_4 SX$	+	$c_4 S'$	+	e_4
5.	SY	=	a_5	+	$b_5 SX$	+	$c_5 S$	+	e_5
6.	GY	=	$a_6\ 1/S'$	+	$b_6 GX$	+	$c_6 G$	+	e_6
7.	GY	=	$a_7\ 1/S'$	+	$b_7 GX$			+	e_7

Based on sample of 500 largest (in terms of total assets) COMPUSTAT firms for 1990. The estimate of the bias in (true standard error of) b, the coefficient on X, SX, or GX is the mean minus 7 (standard deviation) of b estimated over the 250 iterations. $G = S/S'$.

Multiple independent variables

Table 5 presents estimates of coefficient bias and true standard errors from models 1–7 in the previous section, expanded for two independent variables.

We present findings for five cases:

	Corr (S, $X1$) and Corr (S, $X2$)	Corr ($X1$, $X2$)	Corr ($SX1$, S; $SX2$, S)
i	0.0	0.0	equal
ii	0.0	0.5	equal
iii	0.0	0.0	0.8; 0.5
iv	0.0	0.5	0.8; 0.5
v	0.5	implied	equal

As in Table 3, in all cases the correlation between S and S' equals 95 percent, and we report findings for when the standard deviation of $S - S'$ is independent of S, panel A, and proportional to S, panel B. Case i facilitates comparison with the univariate regressions. Cases ii, iii, and iv investigate the findings' sensitivity to assuming $X1$ and $X2$ are correlated and one observed variable, $SX1$, is more highly correlated with the true scale factor, S, than the other, $SX2$. Case v investigates sensitivity to assuming that $X1$ and $X2$ are correlated with the true scale factor. The findings in cases i, ii, and v are insensitive to the magnitude of the correlation between $SX1$ and S and $SX2$ and S; it is 0.854 for the findings reported in Table 5. The term *implied* in case v indicates that we do not calibrate the correlation, rather it is implied by the correlation between S and $X1$ and $X2$; it equals 0.28.[21]

We investigate cases iii and iv where one observed variable is more highly correlated with scale than the other because many extant accounting studies include scale proxies, such as book value of equity, net income, and total assets, as independent variables, but these are not the variables of research interest. For example, Barth's (1994) interest is in the incremental explanatory power of fair values of investment securities and unrealized securities gains and losses; Sougiannis' (1994) interest is in research and development expenditures. Yet, Barth includes as independent variables book value of equity or net income and Sougiannis includes total assets, each of which likely is more highly correlated with the true scale factor than the variables of interest. In Barth (1994), correlations between book value of equity, as $SX1$, and available scale proxies range from 0.79 to 0.98 and between the difference between fair and book values of investment securities, as $SX2$, and the scale proxies range from 0.33 to 0.50. Correlations between net income, as $SX1$, and available scale proxies range from 0.65 to 0.79 and between unrealized securities gains and losses, as $SX2$, and the scale proxies range from 0.44 to 0.47. In our Sougiannis (1994) replication sample, correlations between average R&D expenditures and book value of equity, sales, and total assets are 0.76, 0.76, and 0.70, and correlations between total assets and book value of equity and sales are 0.87 and 0.92.

Consistent with Table 3, Table 5, panel A reveals that when the standard deviation of $S - S'$ is independent of S, regardless of the correlation between S and $X1$ and $X2$, $X1$ and $X2$, and $SX1$ and S, and $SX2$ and S, deflating by a scale proxy, model 3, results in more bias and less estimation efficiency than including the scale proxy as an independent variable, model 4. However, positive correlation between *X1* and *S* and *X2* and *S* results in lower bias for both coefficients than when these variables are uncorrelated (3.212 versus 6.632 and 6.236 for *X1*'s coefficient and 2.884 versus 6.184 and 5.570 for *X2*'s). Perhaps more importantly, when *SX1* is more highly correlated with *S* than *SX2*, the coefficient on *SX2* is much less biased than that on *SX1* (10.919 and 12.675 versus 4.316 and -1.621). When, in addition, *X1* and *X2* are positively correlated, the coefficient on *SX2* essentially is unbiased in all models. This suggests that in studies such as Barth (1994) and Sougiannis (1994), among many others, bias in the coefficient on the variable of interest likely largely is mitigated because another variable more highly correlated with scale is included in the estimation equation.

Table 5, panel B reveals similar findings when the standard deviation of $S - S'$ is proportional to *S*, except that deflation is more effective at reducing bias than in panel A, and when the correlations between *SX1* and *S* and *SX2* and *S* are equal, deflated coefficient estimates have lower standard errors than those from models in which the scale proxy is an independent variable. However, including the scale proxy as an independent variable results in less bias than does deflation.

Heteroscedasticity occurring without coefficient bias

Sections two and three and example 2 in Appendix 2 illustrate that scale differences across sample firms can cause heteroscedasticity in conjunction with scale-related coefficient bias. Example 3 in Appendix 2 illustrates that heteroscedasticity also can occur without scale-related coefficient bias. This section focuses on heteroscedasticity occurring alone not only because it can occur alone, but also because the remedy for scale-related coefficient bias that sections two and three identify as most effective, that is, including a scale proxy as an independent variable, leaves heteroscedasticity as the only remaining problem. Moreover, findings reported in this section indicate that deflation by a scale proxy is not always effective at mitigating heteroscedasticity in otherwise well-specified models, and suggest no reason to believe that deflation by a scale proxy will be more effective in models that have additional econometric problems.

Model and concerns expressed in prior research

We model heteroscedasticity as $Y = a + b\ X + S\ e$ (or $\sqrt{S}e$), where *e* is homoscedastic but the regression error is heteroscedastic. This is the same as the assumed true model in section two, equation 1, except that the error is heteroscedastic. To eliminate scale-related coefficient bias concerns, unlike section two, we assume here that *Y* and *X* are observable. Further details are in Appendix 3. The above relation indicates that, unlike the case of coefficient bias, if deflation is used and heteroscedasticity is the only concern, then the intercept should be deflated along with all other regression variables.[23]

Most empirical accounting studies citing scale differences across firms as a research design issue are concerned with heteroscedasticity (Miller and Modigliani 1966; Bowen 1981; Daley 1984; Olsen 1985; Landsman 1986; Magliolo 1986; Harris and Ohlson 1987; Kormendi and Lipe 1987; Beaver, Eger, Ryan, and Wolfson 1989; Barth, Beaver, and Stinson 1991; Shevlin 1991; Barth 1991, 1994; Barth, Beaver, and Landsman 1992; among others). Both remedies for heteroscedasticity are used often: White (1980) standard errors, which addresses only standard error bias, and deflation, which addresses estimation inefficiency and standard error bias if the scale proxy is "correct." Although Chesher and Jewitt (1987) show that White standard error estimates are biased in small samples, they provide only bounds for the bias. Thus, finite sample properties of White standard error estimates and test for heteroscedasticity in accounting contexts are unexplored. Moreover, the effectiveness of deflating by scale proxies is an open question.

Researchers sometimes cite concern about spurious correlation in connection with using deflators.[24] Deflating by a scale proxy rather than the true scale factor does not induce correlation between regressor and error, and hence does not induce spurious correlation, if the proxy is independent of e.[25] Researchers can investigate this concern by estimating correlations between undeflated regression residuals and candidate deflators. To illustrate, we estimate untabulated correlations between undeflated residuals from regressions of market value of equity on net income, $e1$, and on book value of equity, $e2$, and some common deflators for our COMPUSTAT samples. The deflators are sales, book value of equity, net income, number of shares outstanding, and share price. We find that sales, book value of equity, and price generally are uncorrelated with $e1$ and only price is uncorrelated with $e2$. The others generally are correlated with the residuals. These findings suggest some common deflators can bias coefficient estimates. They also suggest the deflators are omitted variables and therefore should be included in the regression as independent variables.

Evidence on White (1980) standard errors and test for heteroscedasticity, and potential efficiency gains using deflation

We assess the effectiveness of heteroscedasticity remedies by presenting simulation evidence on (1) bias in estimated standard errors, that is, the ratio of estimated to true standard errors in the undeflated model,[26] (2) finite sample properties of White standard errors, that is, ratios of White to true standard errors in the undeflated model and White to true standard errors in the deflated model, (3) efficiency gains from deflating regression variables by the correct, known scale factor to remove heteroscedasticity, that is, ratio of true deflated to true undeflated standard errors, (4) variability of these ratios across samples, and (5) properties of the White test for heteroscedasticity. In the next section, we explore effectiveness of the remedies when the true scale factor is unknown using simulations with five commonly used scale proxies.

The independent variables, X, are net income before extraordinary items, *NI*, and book value of common equity, *BVE*, obtained from COMPUSTAT. The dependent variable is simulated after specifying arbitrary regression coefficients and generating normally distributed errors with mean zero and variances proportional to the independent variable or its square, although we only report findings for the latter.[27] We make these error variance assumptions for three reasons. (1) White (1980) shows that if the error variance weights are independent of X, standard error estimates are unbiased. This suggests that only weights correlated with X are of interest; we investigate two extreme examples. (2) Untabulated findings indicate the assumptions are empirically valid.[28] (3) The examples in Appendix 2 suggest that error variances proportional to X or X^2 are intuitive for merged firms.

Summary statistics for 250 simulation iterations presented in Table 6 indicate that using undeflated variables, OLS standard errors approximate 21 percent of the true standard errors indicating severe standard error bias, yet White standard errors approximate 88 percent of the true ones.[29] However, standard deviations of the White to true undeflated standard error ratio are 26 and 28 percent indicating that, although on average White standard errors approximate true standard errors, the 95 percent confidence limits are 36 (88 - 2*26) and 140 (88 + 2*26) percent, assuming normality.

TABLE 6
Summary statistics from 250 simulation iterations for $Y = a + b\,X + u$, where X = net income or book value of equity, Y is defined as $5 + 5X + e$, and e is drawn from normal distribution with mean zero and variance proportional to X^2. Thus, the true value of b is known to be five.

	X = Net income		X = Book value of equity	
	Mean	sd	Mean	sd
Standard error of coefficient estimate (b):				
estimated / true (undeflated model)	0.205	0.026	0.206	0.027
% estimated < true (undeflated model)	100.0		100.0	
White / true (undeflated model)	0.884	0.261	0.883	0.281
% White < true (undeflated model)	73.2		72.0	
true (deflated) / true (undeflated)	0.184	0.006	0.220	0.007
% true (deflated) < true (undeflated)	100.0		100.0	
White / true (deflated model)				
All	0.991	0.007	0.995	0.023
% White < true (deflated model)	7.2		61.6	
When White X^2 test rejects	0.991	0.007	0.995	0.023
White's X^2 test:				
X^2 (undeflated model)	12.955	5.132	12.547	4.429
% reject	92.8		94.0	
X^2 (deflated model)	2.474	2.301	2.491	2.631
% reject	8.4		10.4	

Based on sample of 500 largest (in terms of total assets) COMPUSTAT firms for 1990. All estimation based on ordinary least squares (OLS). Because the true error variances are simulated and thus known, deflated regressions meet all standard assumptions underlying OLS estimation. Undeflated errors are heteroscedastic.

sd denotes standard deviation, White refers to White (1980).

As expected, Table 6 indicates that estimation using variables deflated by the *true* scale factor, and thus with homoscedastic errors, always yields efficiency gains. Deflated estimates are 4.5 and 5.4 times as efficient as undeflated estimates (1/0.220 to 1/0.184). Small standard deviations of the ratios (0.006 and 0.007) indicate that confidence limits for these estimates are quite narrow. White standard errors for the deflated regressions are almost identical to the true deflated standard errors with mean ratios of 0.991 and 0.995. This suggests using White standard errors is appropriate even with homoscedastic errors. Moreover, although one would expect White standard errors to vary considerably from sample to sample because they are based on estimated residuals rather than the identity matrix, standard deviations for these ratios are small (0.007 and 0.023). Untabulated findings indicate the ratios' standard deviations remain small even when the error term variance increases and thus the regression R^2 decreases.

Because our simulated regressions are well-specified, the White χ^2 statistic in Table 6 represents a test of only heteroscedasticity. The findings indicate the test is quite successful in detecting heteroscedasticity when it exists; it does so in 92.8 and 94.0 percent of the iterations. But in the 100-firm subsample, the rejection rate is only 57 percent—despite our extreme heteroscedasticity assumption. This indicates that the commonly followed procedure of using White standard errors only if the White χ^2 test rejects homoscedasticity can lead to incorrect inferences from biased standard error estimates almost 43 percent (100 - 57) of the time in smaller samples, although it does so just 6 to 7 percent of the time in larger samples. Because we also find that White standard errors are reasonably accurate even with homoscedasticity, this finding reinforces our suggestion that researchers base inferences on White standard errors regardless of whether the White test rejects homoscedasticity. In 8.4 and 10.4 percent of the iterations, test statistics for deflated regressions reject at the five percent significance level the null of homoscedasticity when it is true. Thus, the White test appears slightly conservative in indicating heteroscedasticity.

Evidence on deflation using a scale proxy

Next we explore effectiveness of deflation as a remedy for heteroscedasticity when the true scale factor is unknown and regression variables are deflated by a scale proxy. Table 7 presents summary statistics from deflated estimation simulations when the error variances are simulated to be proportional to X^2, where $X = NI$ or BVE, and several common deflators are scale proxies. We con-

TABLE 7
Means from 250 simulation iterations for $Y/S' = a/S' + b\,X/S' + u$, where X = net income or book value of equity, Y is defined as $5 + 5X + e$, and e is drawn from normal distribution with mean zero and variance proportional to X^2. S' is a deflator used to mitigate e's heteroscedasticity. Five commonly-used deflators are considered: total assets, *TA*, sales, *SALES*, book value of equity, *BVE* or net income, *NI* (when not equal to *X*), number of shares outstanding, *NUMSHR*, and share price, *PRICE*.

Deflator:	*TA*	*SALES*	*BVE* or *NI*	*NUMSHR*	*PRICE*
Standard error of *b*:					
true deflated / true undeflated: *Measures true efficiency gain (ratio<1) or loss (ratio>1)*					
X = *NI*	0.549	0.402	1.165	3.857	0.549
X = *BVE*	0.354	0.502	2.277	3.897	0.583
White deflated / White undeflated: *Estimate of efficiency gain (ratio<1) or loss (ratio>1)*					
X = *NI*	0.670	0.491	1.095	0.260	0.647
X = *BVE*	0.446	0.600	1.848	0.094	0.686
estimated deflated / true deflated: *Measures accuracy of estimated deflated standard errors (ratio=1 if accurate)*					
X = *NI*	0.427	0.518	0.163	0.023	0.317
X = *BVE*	0.735	0.435	0.090	0.012	0.297
White deflated / true deflated: *Measures accuracy of White deflated standard errors (ratio=1 if accurate)*					
X = *NI*	0.954	0.945	0.737	0.052	0.943
X = *BVE*	0.989	0.921	0.613	0.018	0.937
% White test rejections:					
Indicates whether deflation eliminated heteroscedasticity (low ratio indicates elimination or mitigation)					
X = *NI*	99.6	91.6	29.6	10.0	80.0
X = *BVE*	100.0	62.4	32.4	14.8	81.2

Based on sample of 500 largest (in terms of total assets) COMPUSTAT firms for 1990. All estimation based on ordinary least squares (OLS). Because the true error variances are simulated and thus known, deflated regressions do not necessarily meet all standard assumptions underlying OLS estimation. Undeflated errors are heteroscedastic. White refers to White (1980) standard error estimates and test for heteroscedasticity.

sider as deflators total assets, *TA*, sales, *SALES*, share price, *PRICE*, number of shares outstanding, *NUMSHR*, and either *BVE* or *NI* when it does not equal *X*. Because simulated regression errors are independent of the deflators, deflation does not induce spurious correlation.

The Table 7 findings indicate that efficiency gains from deflation in Table 6 are not always achieved when scale proxies are used. The ratio of true deflated to true undeflated standard errors is less than one in only six of the 10 instances. We find that efficiency *loss* using deflation can be almost 300 percent. In unsimulated estimation settings, true standard errors are unknown and efficiency gain or loss only can be estimated. The Table 6 findings that White standard errors generally are close to the true ones suggest that one method for estimating efficiency gain or loss is to compare White standard errors for the deflated regression with those for the undeflated regression. Table 7 confirms that this provides reasonable approximations to the corresponding ratio of true standard errors except when *NUMSHR* is the deflator.[30] Using *NUMSHR* as the deflator results in efficiency losses, even though White standard error estimates indicate efficiency *gains* and the White test indicates *NUMSHR* is the deflator most effective at mitigating heteroscedasticity.

Table 7 indicates OLS estimated deflated standard errors always understate true ones—the understatement ranges from 26.5 to 98.8 percent (1– 0.735 to 1-0.012). White deflated standard errors also often understate true ones, although they are much closer except when *NUMSHR* is the deflator. Table 7 also indicates the White test often rejects the null of homoscedasticity for deflated regressions, particularly when *TA*, *SALES*, or *PRICE* is the deflator, even though estimation efficiency is increased. Together with the findings in Table 6, this indicates deflation does not always eliminate heteroscedasticity, at least the form of heteroscedasticity the White test identifies. It identifies only heteroscedasticity related to the independent variables and their cross-products because only this type of heteroscedasticity results in biased standard error estimates. It is possible that although this form of heteroscedasticity has not been eliminated by deflation, other forms have been mitigated and thus estimation efficiency has increased.

Illustrations using recent studies

Kothari and Zimmerman (1995) use number of shares as a deflator. Table 7 suggests that although the White test indicates *NUMSHR* deflation is effective in mitigating heteroscedasticity, it greatly increases estimation inefficiency and White standard errors in *NUMSHR* deflated specifications substantially understate the true ones. Table 4, panel A indicates that in all undeflated regressions the White (1980) χ^2 test rejects at very low significance levels the null of homoscedasticity. However, apparently share deflation does not eliminate heteroscedasticity (models 5 and 6, White χ^2 p value = 0.0297 and 0.0070). Moreover, consistent with findings in Table 7, White standard errors for ERC estimates in undeflated specifications are smaller than those in deflated specifications, indicating efficiency *losses* from deflation by *NUMSHR.*

Sougiannis (1994) deflates by total assets, which Table 7 indicates reduces heteroscedasticity in our samples. However, Table 4, panel C reveals that the White χ^2 test rejects the null of homoscedasticity in all specifications with p values ranging from 0.0020 to 0.0060. For the larger sample, the test statistic is larger for the deflated specification, model 4, than the undeflated one, model 3, although coefficient standard errors are smaller in the deflated regressions indicating an increase in estimation efficiency. Yet, for the smaller sample, White standard errors for the deflated regression, model 2, are not substantially smaller than the undeflated ones, model 1, indicating efficiency gains from deflation are minimal. It appears deflation by total assets does not noticeably reduce heteroscedasticity in these specifications.

Summary and concluding remarks

This study investigates two major econometric issues commonly arising in empirical accounting research studies that use cross-sectional levels-based research designs: coefficient bias and heteroscedasticity. These issues arise because regression variables in such research designs likely are affected by cross-sectional scale, or size, differences among sample firms. That is, large firms have large values of most variables and small firms have small values, and these magnitude differences often are unrelated to the research question.

We provide evidence on the extent of scale-related econometric problems in accounting research contexts and the effectiveness of available remedies: deflating regression variables by a scale proxy, including a scale proxy as an independent variable, and using White (1980) heteroscedasticity-consistent standard error estimates. Because tractability limits our analytical development to regression models with a single independent variable and to situations in which the scale factor and true independent variable are uncorrelated, we use simulations to relax these restrictions. We base our simulations on COMPUSTAT firms and accounting data to mimic samples, empirical distributions of accounting variables, and estimation equation specifications typical in empirical financial accounting research. Although our analytical development is general, our selection of accounting and scale variables is motivated by regressions of, for example, market value of equity on net income or book value of equity.

The existence of omitted variables coefficient bias and heteroscedasticity is well known. However, we develop expressions for coefficient bias that permit us to identify factors affecting the bias, and for heteroscedasticity-related standard error bias that permit us to estimate it from observed variables. We model coefficient bias as an omitted variable and show the bias depends on the intercept in the true regression and on the coefficients of variation of the independent variable and scale. Because scale is an omitted variable, coefficient bias can be mitigated by including a scale proxy as an independent variable. However, researchers often deflate regression variables by scale proxies because deflation can mitigate heteroscedasticity as well as coefficient bias. Consequently, we investigate whether deflation by a scale proxy or its inclusion as an independent variable is more effective at reducing coefficient bias. We also provide empirical evidence on properties of White (1980) standard error estimates and test for heteroscedasticity, and efficiency effects of deflation. Our analysis suggests diagnostics and other techniques researchers can use to evaluate the extent of coefficient bias and heteroscedasticity they face, and the effectiveness of the remedies they select to mitigate them.

Regarding coefficient bias, we find the following. (1) Scale proxies commonly used to deflate regression variables—net income, book value of equity, sales, total assets, number of shares, and share price—typically mitigate coefficient bias only by a small amount, if at all. In some cases, deflation worsens bias. (2) Surprisingly, proxies 95 percent correlated with the true scale factors can *worsen* bias if used as deflators but are quite effective at mitigating bias if included as independent variables, as are proxies less highly correlated. (3) With two independent variables, the coefficient on the variable more (less) highly correlated with scale is more (less) biased. Our inferences regarding coefficient bias are insensitive to correlation between the true scale factor and independent variables and multiple independent, potentially correlated, variables.

Regarding heteroscedasticity, we find the following. (1) In undeflated specifications, OLS-estimated standard errors can severely underestimate the true ones; yet, White (1980) standard errors are close to the true ones regardless of whether errors are heteroscedastic. (2) In deflated specifications using some deflators, notably number of shares, White standard errors also can severely understate the true ones. (3) Deflation by scale proxies does not always eliminate heteroscedasticity as indicated by the White χ^2 test and can result in efficiency *losses* of up to 300 percent. Efficiency losses can result even when the deflator is 95 percent correlated with the true scale factor. (4) White's test is effective at identifying heteroscedasticity in our larger samples when regressions are otherwise well-specified, although it is somewhat conservative.

We estimate specifications similar to those in Kothari and Zimmerman (1995), Barth (1994), and Sougiannis (1994), to illustrate the implications of our findings. We find that although none of these studies' reported inferences are attributable to scale differences, our simulation findings are confirmed. (1) Deflation by some deflators in related samples does not mitigate substantially coefficient bias, can result in incorrect inferences, does not always eliminate heteroscedasticity as indicated by the White test, and can decrease estimation efficiency. (2) Findings can be sensitive to the deflator choice. (3) Using a scale proxy as an independent variable rather than as a deflator is effective at mitigating coefficient bias. (4) When the variable of interest is less highly correlated with scale than other independent variables, any scale-related coefficient bias is mitigated. We also provide evidence that the deflator's coefficient of variation is a factor in determining coefficient bias and that our diagnostic for identifying coefficient bias is effective.

In summary, our findings suggest that if scale differences are of concern to empirical accounting researchers, the most effective remedy is to include a scale proxy as an independent variable and report inferences based on White standard errors. Deflation has unpredictable effects on coefficient bias, heteroscedasticity, and estimation efficiency. Our findings also suggest that in multiple regression specifications, bias in the coefficient on the variable of interest, for example, components of net income or particular assets or liabilities, such as securities gains and losses or fair value of investment securities, is less of a concern when estimation equations also include a variable more highly correlated with scale such as net income or total assets.

Appendix 1

This appendix describes procedures we use to develop estimation equations and samples similar to those in Kothari and Zimmerman (1995), Barth (1994), and Sougiannis (1994).

Kothari and Zimmerman (1995) estimate earnings response coefficients, ERC, by estimating various specifications of the relation between earnings and share prices, including share price regressed on earnings per share, *EPS*, ("price model"). Kothari and Zimmerman deflate all variables by number of shares to mitigate heteroscedasticity. We estimate Kothari and Zimmerman's price model, undeflated and deflated by number of shares.

$$MVE = a + b\,NI + e \qquad \text{(KZ1)}$$

and

$$PRICE = a' + b'\,EPS + e' \qquad \text{(KZ2)}$$

We also estimate versions of equations KZ1 and KZ2 that include scale proxies as additional independent variables.[31] To approximate Kothari and Zimmerman's annual sample we use all COMPUSTAT firms with non-missing data and non-negative net income for 1989 after eliminating observations with *MVE*, *PRICE*, *NI*, or *EPS* greater than five standard deviations from their mean (nobs = 1,906).[32]

Barth (1994) investigates value-relevance of disclosed fair value estimates of banks' investment securities by regressing market value of equity on book value of equity and investment securities' fair value, and reports as a sensitivity check a regression of market value of equity on net income and fair value securities gains and losses. Using Barth's data, we estimate three versions of two estimation equations similar to those in Barth (1994).[32]

$$MVE = c + d\,FVBV + u, \qquad \text{(B1)}$$

and

$$MVE = c' + d'\,URSGL + u'. \qquad \text{(B2)}$$

Equation B1 includes as an independent variable the difference between fair and book values of investment securities, *FVBV*, and equation B2 includes unrealized securities gains and losses, *URSGL*.

Sougiannis (1994) examines whether net income reflects the benefits of past research and development (R&D) expenditures by regressing net income on total assets and current and lagged R&D expenditures, with all regression variables deflated by total assets. We replicate Sougiannis' regressions of net income on total assets and current and lagged R&D expenditures using 1989 COMPUSTAT data. For simplicity, we use as our R&D variable average R&D over seven years, rather than Sougiannis' variable obtained by fitting an Almon lag over R&D variables lagged up to seven years. To obtain a sample size comparable to Sougiannis, we include all firms with assets greater than $1 billion with available R&D expenditure data. This results in a sample of 266 firms, compared with Sougiannis' sample of 311 firms for his latest sample year, 1985. We estimate

$$NI = f + g\,TA + h\,R\&D + v\,, \tag{S1}$$

and

$$(NI\,/\,TA) = g' + f'(1\,/\,TA) + h'\,(R\&D\,/\,TA) + v'\,. \tag{S2}$$

Table 4, panel C reveals that in the undeflated regression, the coefficient on *R&D* of 1.65 is comparable to Sougiannis' mean coefficient summed across seven lags of 2.08, providing some evidence that using average R&D does not invalidate our comparison with Sougiannis' findings.

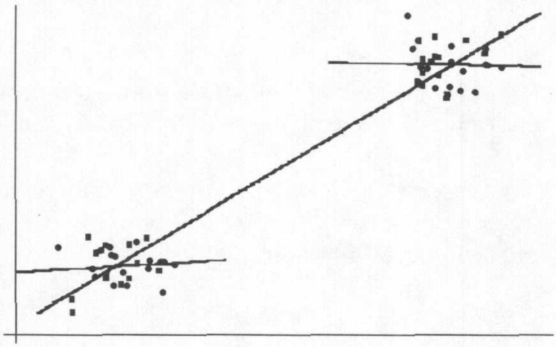
Appendix 2

This appendix provides three examples to illustrate scale-related problems. Example 1 illustrates how coefficient bias results from scale differences, example 2 illustrates a case when both coefficient bias and heteroscedasticity are observed, and example 3 illustrates a case when scale differences cause heteroscedasticity but not coefficient bias.

Example 1

For simplicity, assume the scale factor has two values: large and small. Within each scale group there is no relation between variables Y and X, but both variables are larger for large firms than for small firms. One can think of these groups as money-center and regional banks, and of market values as Y and depreciation as X. Because banks' profits depend on their financial assets and liabilities rather than on depreciable assets, there is no relation between Y and X. Figure A1 depicts this situation. The short lines representing the regressions of Y on X for each group are nearly horizontal, consistent with no relation between Y and X. However, consistent with scale differences between the groups, the intercepts of the two regression lines differ. Thus, when Y is regressed on X for both groups combined, the slope coefficient is positive—a biased estimate resulting from scale differences between the two groups. In this example, coefficient bias occurs without true heteroscedasticity because the variables are generated with homoscedastic errors.

Figure A1: Illustration of coefficient bias based on simulated data



For points in the lower left and upper right corners, Y is independent of X. For points in the upper right corner, both X and Y have means equal to 20 and for points in the lower left corner, they have means equal to five. Because X and Y are independent, the regression line for each group is nearly horizontal. However, the regression lines do not have the same intercept. Thus, the regression line is not horizontal when estimated for both groups combined. The intercept difference between the two groups results in a biased slope coefficient in the combined regression.

Example 2

Assume a researcher plans to estimate a cross-sectional regression equation: $MVE = a + b\ NI + e$, based on the constant P/E ratio valuation model where MVE is market value of equity and NI is the firm's earnings. To introduce scale, think of the above relation as being descriptive of unit firms with scale equal to one, identified by the subscript i, and large firms as mergers of unit firms. Let $i = 1$ represent the small firm, and let the merger of two unit firms, $i = 2$ and 3, represent the large firm whose scale equals two. Thus, the equity value of the small firm is $MVE_S = MVE_1 = a + b\ NI_1 + e_1$ and of the large firm is $MVE_L = (MVE_2 + MVE_3) = 2\ a + b\ (NI_2 + NI_3) + (e_2 + e_3)$. In the regression equation of MVE on NI, scale is an omitted explanatory variable with values equaling one and two for the small and large firms. Its omission results in a biased estimator of b.[34] Also, if the variances of e_i are equal for all i, the variances of e_S and e_L differ, that is, the errors are heteroscedastic.[35]

Example 3

To guarantee the absence of coefficient bias assume, consistent with a cross-sectionally constant P/E ratio, that the intercept, a, equals zero. That is, $MVE_L = (MVE_2 + MVE_3) = b\ (NI_2 + NI_3) + (e_2 + e_3)$ and $MVE_S = MVE_1 = b\ NI_1 + e_1$. Here, the regression of MVE on NI has no omitted variable. However, $e_L = e_2 + e_3$ and $e_S = e_1$. As in the previous example, the variances of e_L and e_S differ causing heteroscedasticity. Note that if e_2 and e_3 are independent, then the error terms' variances are proportional to the scale factor. If they are perfectly correlated, the variances are proportional to the square of the scale factor. One can think of the independence case as a conglomerate firm and the perfect correlation case as a single product firm.

Appendix 3

This appendix develops expressions for the ratio of the estimated standard error variance to the true variance as a function of the moments of the scale factor's distribution.

In the standard well-specified regression equation $Y = a + b X + e$ with $Var(e_i) = w_i$ indicating heteroscedasticity, it is well known that the expectation of the ratio of the estimated variance to the true variance of $\hat{b}$ is

$$\frac{\sum w_i \sum x_i^2 - \sum w_i x_i^2}{n \sum w_i x_i^2}, \tag{A1}$$

where x_i equals $X_i - \overline{X}$, $\overline{X}$ is the mean of X, and n is the sample size.

White (1980) shows that if the weights are independent of X then the expectation of this ratio is one: there is no bias in the variance estimate. Negative correlation between w and X results in a ratio greater than one indicating that the estimate overstates the true standard errors. However, likely the more common situation in accounting contexts is when w and X are positively correlated and hence the estimate understates the true standard errors.

To obtain insight into the factors affecting the magnitude of the standard error bias, we assume that the scale factor, S, is the independent variable, X, because this is an extreme case—correlation between X and S equals one. As indicated in Appendix 2 and section three, scale-related heteroscedasticity likely results in weights proportional to the scale factor, S, or S^2. It can be shown that when w_i is proportional to X_i^2 or, equivalently, the error term standard deviations are proportional to X_i, the ratio of estimated variance to true variance equals[36]

$$\frac{n-1}{n} + \frac{Var - Ku \cdot Var - 2sd \cdot Sk \cdot \overline{X}}{Ku \cdot Var + 2sd \cdot Sk \cdot \overline{X} + \overline{X}^2}, \tag{A2}$$

where Ku is kurtosis $((\Sigma x^{i}_{4}/n)/Var^2)$, Sk is skewness $((\Sigma x_i^3/n)/Var^{3/2})$, sd is standard deviation of X ($\sqrt{\text{Var}}$), and Var is variance of X $(\Sigma x_i^2/n)$.

Expressing the bias ratio as a function of the moments of X facilitates understanding of sources of bias. For example, the equation shows that the higher the skewness and kurtosis, the greater the downward bias in the standard error estimate. Also, downward bias is greater when $\overline{X}$ is small compared to X's standard deviation. The bias ratio for normally distributed X where $Sk = 0$ and $Ku = 3$ in the worst case, when $\overline{X}$ equals zero, equals one-third indicating that estimated standard errors are 0.58 ($1/\sqrt{3}$) times the true standard errors. In our sample, when X is net income, *NI*, and book value of equity, *BVE*, the standard error ratios, that is, the square root of equation A2, are 0.207 and 0.208

which are lower than when X is normally distributed indicating that bias is more severe in observed data.

When the error term standard deviations are proportional to $\sqrt{X}$ and thus w_i is proportional to X_i, the ratio of OLS variance to true variance equals

$$\frac{\overline{X}}{sd \cdot Sk + \overline{X}} - \frac{1}{n} \quad \text{(A3)}$$

Equation A3 shows that if the skewness of X does not equal zero the ratio differs from one, but if $\overline{X}$ is much larger than $Sk \cdot sd$ and hence $\overline{X}$ approximately equals $Sk \cdot sd + \overline{X}$, then the ratio is close to one.[37] When X equals *NI* and *BVE,* the standard error ratios for our sample are 0.364 and 0.372 indicating that the bias here is less severe than when heteroscedasticity weights are proportional to X^2.

Endnotes

1 We do not address whether returns- or levels-based designs are appropriate. Two advantages of returns- or first-difference-based designs are that heteroscedasticity largely is avoided and effects of intertemporally constant omitted variables, including scale, are eliminated. If scale is intertemporally constant, it also can be eliminated by estimating firm-specific intercepts, with a sufficient number of time-series observations. However, Landsman and Magliolo (1988) show that levels models can dominate returns models, for example, when model parameters or omitted variables are not intertemporally constant. Moreover, returns-based designs and estimating firm-specific intercepts are not always feasible, such as when only a few years' data are available (e.g., Barth, Beaver, and Landsman 1996) or the variable of interest varies little over time (e.g., Guenther and Trombley 1994; Barth and McNichols 1994; among others). Returns designs also are problematic if when the variable's information is reflected in share prices is unknown (e.g., Amir, Harris, and Venuti 1993). Moreover, Kothari and Zimmerman (1995) show that because prices lead accounting recognition in incorporating new information, price-levels designs provide more economically sensible results.

2 Scale issues also are relevant to finance and economics research, among others. Bernard (1987) also examines model specification issues related to scale in cross-sectional accounting capital markets research. However, our research objectives and questions differ from his. Others (e.g., Kuh and Meyer 1955; Lev and Sunder 1979) examine ratios as a control for firm size. However, Kuh and Meyer analyze correlation, not regression, coefficients and thus their findings provide limited insights about using regression techniques.

3 We use three studies because not all of our findings are easily illustrated using one. Moreover, they illustrate application of our findings in different settings. We select Kothari and Zimmerman (1995) because (1) it analyzes the advantages of levels-based research designs. (2) The deflator is number of shares, which we identify as potentially problematic. (3) The estimation equation has only one independent variable, as in our analytical development. We select Barth (1994) and Sougiannis (1994) because (1) they are recent examples of levels-based studies that estimate different equation specifications, all common to accounting research. Barth's (1994) specifications are based on asset and liability or net income valuation models and Sougiannis' (1994) are based on Ohlson (1989). (2)

Both use multiple regressions with the variable of interest likely less correlated with scale than other independent variables. (3) Both use deflation, but the deflators differ. Barth (1994) uses number of shares, and Sougiannis (1994) uses total assets. Moreover, data similar to that used in all three studies is available to us.

4 Christie (1987) notes that if observed variables are not homogeneous of degree one in the scale factor, then deflation does not eliminate coefficient bias. Our findings suggest that even if observed variables are homogeneous of degree one in the scale factor *and* a highly correlated scale proxy is available, deflation does not mitigate coefficient bias.

5 Landsman and Magliolo (1988) present a different perspective. They argue that because economic earnings X^* the true variable, is unobservable, any variable correlated with earnings can be a measure of X^*. They also caution that deflation can eliminate the effect of research interest; here, deflation by sales can eliminate *DEPR*'s correlation with X^*. Although Landsman and Magliolo's interpretation is possible, our evidence is consistent with coefficient bias.

6 More generally, observed variables X_s and Y_s could equal $k + S_iX_i$ and $k' + S_iY_i$. Then, the relation is $Y_s = (k' - b_{Y.X}\, k) + a\, S_i + b_{Y.X}\, X_s + (S_ie_i)$ and regression with observed variables has an intercept in addition to the other variables in equation 2.

7 The reverse also can hold, that is, e's variance can be such that e is heteroscedastic but Se is not. The model's intuition easily can be applied to this alternative situation.

8 Table 1, panel B reports $\bar{S}^2/Var(S)$ for several accounting variables commonly used as scale proxies, that is, total assets, sales, book value of equity, and net income, for a sample of COMPUSTAT firms.

9 Consistent with Christie (1987) and Landsman and Magliolo (1988), equation 2 also shows that if the scale factor and its coefficient are intertemporally constant, first-differencing eliminates aS, permitting an unbiased estimate of $b_{Y.X}$. However, first-differencing can reduce variation in X, the variable of interest, decreasing efficiency of estimating $b_{Y.X}$ (Landsman and Magliolo). Moreover, scale or its coefficient may not be intertemporally constant. See endnote 5.

10 The smallest bias ratio is 28 percent in the 100-firm sample, indicating improvement of 72 percent, when *BVE* is the true scale factor and *TA* is the deflator.

11 If the relation is proportional to the scale factor, as assumed in the analytical development, then one also can include a scale proxy as an independent variable and test whether its coefficient equals zero.

12 S' is constant across simulation iterations. Repeating our simulations with different assumptions yields findings similar to those reported: (1) S' varying across iterations, (2) v uniformly distributed over [-2,635, 2,635], which is required so that the correlation of S and S' equals 0.95, and (3) net income as the true scale factor. Also, findings based on 30 and 100 simulation iterations are similar to those we tabulate, which are based on 250 iterations, suggesting that our conclusions likely would be unaltered with a larger number of iterations.

13 Including an intercept in models 4 and 5 does not follow directly from equation 2. Estimating the two models without an intercept yields findings similar to those from estimating them with an intercept reported in Table 3. We include an intercept in these two models because researchers often include intercepts and an intercept would be required for correct model specification if the means of S and S' differed. In our simulations, the means are equal.

14 The intuitive reason why the deflator performs poorly when $s(S - S')$ is independent of S is that under that assumption, the magnitude of $S - S'$ is the same regardless of the magnitude of S. When S's coefficient of variation is high, the average magnitude of $S - S'$ is large (small) relative to small (large) values of S, even if S' is 95 percent correlated with S. Thus, S' performs poorly as a deflator.

15 White standard errors in models 1, 4, and 5 approximate 98, 84, and 78 percent of the true ones, which is consistent with findings in section four where we investigate remedies for heteroscedasticity unaccompanied by coefficient bias. However, in model 6 they approximate only 39 percent. In general, White standard errors are noticeably less accurate in deflated models.

16 When S' is included as an independent variable, the plim of the bias in b is:

$$\frac{a\overline{X}[1-(Corr(S,S'))^2]}{[Var(X)(1+\overline{S}^2/Var(S))]+[1-(Corr(S,S'))^2][\overline{X}^2+Var(X)]}.$$

As indicated above, when S' is a deflator the bias in b depends on the moments of G which cannot be characterized in terms of the distributions of S and S'. Thus, we do not present a formula for the difference in bias between the two alternatives and base our analysis on simulations.

17 Our model 5 ERC estimate, 2.93, differs substantially from Kothari and Zimmerman's (1995), 6.55. One possible explanation is that, consistent with table 7, the true standard errors are much larger than those estimated, and Kothari and Zimmerman's sample of 38,890 observations results in more estimation efficiency than our sample of 1,906. However, sample sizes as large as Kothari and Zimmerman's are not common in accounting research.

18 Inferences are unaltered when book value of equity or total assets (net income or total assets) is an additional independent variable in model 5 (2).

19 That coefficient bias attributable to scale is more problematic in the securities gains and losses specifications, models 4–6, than in the investment securities specifications, models 1–3, is not unexpected given the correlations among the variables. Untabulated correlations indicate book value of equity and net income are more highly correlated with the scale proxies, book value of equity, net income, operating revenue, and total assets, than are the difference between fair and book values of investment securities and unrealized securities gains and losses. However, correlations between book value of equity and the scale proxies range from 1.87 to 2.42 times those of the difference between fair and book values of investment securities, whereas correlations between net income and the scale proxies are only 1.49 to 1.69 times greater than those of unrealized securities gains and losses.

20 We cannot explain why we observe coefficient bias for the larger, but not smaller, sample. However, section two suggests that the deflator's coefficient of variation affects bias. In the larger sample, the deflator's, *TA*, coefficient of variation is 3.56, which is higher than that in the smaller sample, 1.92. However, as noted in section two, the deflated coefficient's bias is complex and coefficients of variation are not unambiguous indicators of bias severity, particularly across samples. For example, in the Barth (1994) sample, the deflator's, *NI*, coefficient of variation is lower than that of *TA* in the smaller Sougiannis (1994) sample, 1.30, and yet in the Barth application, using *NI* as a deflator appears to result in coefficient bias.

21 *X1* and *X2* are simulated as follows for cases i–v.

i $X1$ = 50*Z2 + 100; $X2$ = 50*Z3 + 100.

ii $X1$ = 35.35*Z1 + 35.35*Z2 +100; $X2$ = 35.35*Z1 + 35.35*Z3 +100.
iii $X1$ = 50*Z2 + 80.896; $X2$ = 50*Z3 + 35.
iv $X1$ = 35.35*Z1 + 35.35*Z2 + 80.896; $X2$ = 35.35*Z1 + 35.35*Z3 + 35.
v $X1$ = 0.005402*(*BVE* - 3,181) + 43.301*Z2 + 100; $X2$ = 0.005402*(*BVE* - 3,181) + 43.301*Z3 + 100, where Z1–Z3 are independent standard normal random variables.

22 We estimate models 6 and 7 to provide some evidence on this question. Model 6 (model 7) includes the true scale factor (a scale proxy) as an independent variable to mitigate coefficient bias and deflates by a scale proxy to mitigate heteroscedasticity. The findings reported there indicate that deflation by a scale proxy is not effective at mitigating heteroscedasticity, except in model 6 when the standard deviation of $S - S'$ is proportional to S.

23 Note that using weighted least squares, WLS, and deflating the dependent variable and all independent variables including the intercept are equivalent. These techniques are potential remedies for heteroscedasticity but are not remedies for scale-related coefficient bias.

24 For example, Miller and Modigliani (1966) reject one deflator, after-tax firm value, *V-tD*, as likely to introduce spurious correlation but accept another, total assets. However as Lev and Sunder (1979) clarify, deflation does not cause spurious correlation simply because the same variable appears in the denominator on both sides of the regression equation. A regression coefficient is biased only if the regressor is correlated with the error. Miller and Modigliani reject *V-tD* because it is the dependent variable and hence correlated with the undeflated error. Christie's (1987) statement that any deflator other than a function of the independent variables potentially introduces specification error suggests the same reasoning. Findings sensitive to deflator choice suggest potential for spurious correlation (e.g., Lustgarten 1982; Olsen 1985).

25 To see that Se/S' is uncorrelated with X/S' if e is independent of S and S' (and therefore also $1/S'$), let $S'' = 1/S'$. Then $\text{Cov}(S''Se, S''X)$

$= \text{E}((S''Se - \text{E}(S''Se))(S''X - \text{E}(S''X))$

$= \text{E}((S''Se)(S''X))$ because $\text{E}(S''Se) = \text{E}(S''S)\text{E}(e) = 0$

$= \text{E}(e)\text{E}(S''^2SX)$ because $\text{E}(XY) = \text{E}(X)\text{E}(Y)$ if X and Y are independent

$= 0$ because $\text{E}(e) = 0$.

Our simulations confirm the unbiasedness of coefficient estimates in regressions using deflated variables. Correlations between S' and higher moments of e can induce econometric problems other than coefficient bias, such as heteroscedasticity.

26 We use simulations to assess the ratio of estimated to true standard errors because although the ratios can be estimated using formulas developed in Appendix 3, their variability cannot. Christie (1987, section four) provides evidence on understatement of reported standard errors by relating time series variation of reported coefficient estimates to mean reported standard errors from successive cross-sectional estimations. He also finds that reported standard errors can severely understate true standard errors. However, his procedure requires several time series observations and assumptions about coefficients' time series properties. Also, Griliches and Hausman (1986) show how exploiting panel data can mitigate effects of omitted variables such as scale. We focus on only one cross-sectional regression and do not explore the Griliches and Hausman techniques because time series properties of estimators are beyond this study's scope.

27 Findings with error variances proportional to X are similar to those reported except, as expected from estimations reported in Appendix 3, differences among

estimates usually are smaller in the untabulated simulations. As in section three, we base all simulations on four COMPUSTAT samples. Although we report summary statistics for only the 500-firm sample, untabulated summary statistics for the other samples are similar to those reported unless noted.

28 To assess validity, we estimated market value of equity on book value of equity and on net income for our four COMPUSTAT samples. In six of eight cases, the assumption that the weights are proportional either to X or X^2 (tested by regressing the logarithm of squared residuals on X and the squared logarithm of X, [Park 1966]) cannot be rejected.

29 For the 100-firm sample, White standard errors are only about 60 percent of true standard errors. The relatively poor performance of White standard errors in this sample largely is attributable to variation in scale within the sample and not to small sample size. Recall that the 100-firm sample is selected randomly from the 500-firm sample. When we repeat the simulations on the smallest and largest 100 firms in the 500-firm sample, White standard errors are much closer to true standard errors. In the sample of smallest firms, the mean ratios are 0.925 and 0.972 when $X = NI$ and $X = BVE$. For the largest firms, they are 0.870 and 0.852.

30 For the 100-firm sample, the ratio of White deflated to White undeflated standard errors consistently exceeds the corresponding ratio of true standard errors unless *NUMSHR* is the deflator, indicating that White standard error ratios generally understate efficiency gains for the smallest sample.

31 We estimate, but do not tabulate, similar equations using total assets as a scale proxy. Its coefficient insignificantly differs from zero in all specifications.

32 We eliminate 32 observations with a noticeable effect on the reported findings. However, findings from eliminating observations greater than three standard deviations from their mean are similar to those reported.

33 Barth's analogous regressions include as independent variables book values of equity before investment securities and investment securities (net income before securities gains and losses and realized securities gains and losses). For the sake of parsimony, we combine these into a single variable, book value of equity (net income).

34 If sample firms each are formed by merging a varying number of firms, scale will have a range of values, not only two. Also, we do not consider situations when the coefficient, b, varies cross-sectionally with scale, for example, if factors such as risk, growth, and tax status affect b, and are linearly related to scale. In such cases, the correctly-specified regression equation is different from the one described above.

35 To see that scale *differences* among sample observations cause coefficient bias and heteroscedasticity, consider two large firms each comprising two small (unit) firms, for example, merge firm $i = 1$ with $i = 2$ and $i = 3$ with $i = 4$. These firms' equity values can be expressed as $(MVE_i + MVE_{i+1}) = 2a + b\,(NI_i + NI_{i+1}) + (e_i + e_{i+1})$, $i = 1$ and 3. Each has scale of two. Estimating this regression yields an unbiased estimate of b, and an intercept whose expectation equals $2a$. That the estimated intercept is $2a$ rather than a is of concern only if hypotheses are about the intercept's magnitude. Moreover, the errors are homoscedastic if the e_i's have equal variance, and the covariance of e_1 and e_2 equals that of e_3 and e_4. Assuming the observed variable equals $S{\bullet}X$ is equivalent to assuming that the unit firms' X values are perfectly correlated. If, instead, the correlation is zero, then the bias likely will be smaller than that calculated from equation 4. Also, zero correlation between unit firms' error terms causes the error variances in the regression using observed variables to be proportional to S rather than S^2. In that

case, deflating observed variables—dependent and independent—by S overcorrects for heteroscedasticity.

36 To derive equation A2 set $w_i = k X_i^2 = k (x_i + \overline{X})^2$ in equation A1, and use formulas for variance, skewness, and kurtosis.

37 The comparative statics assume changes in one moment of the distribution do not affect other moments, which is not valid for many common distributions. In such cases, interdependencies need to be considered in determining effects on the ratio of estimated to true variance of changes in moments.

References

Amir, E., T.S. Harris, and E.K. Venuti. A comparison of the value-relevance of U.S. versus non-U.S. GAAP accounting measures using form 20-F reconciliations. Supplement to the *Journal of Accounting Research* (1993), 230–264.

Barth, M.E. Fair value accounting: Evidence from investment securities and the market valuation of banks. *The Accounting Review* (1994), 1–25.

————-. Relative measurement errors among alternative pension asset and liability measures. *The Accounting Review* (1991), 433–463.

————-, W.H. Beaver, and W. R. Landsman. Value relevance of banks' fair value disclosures under SFAS 107. *The Accounting Review* (forthcoming October 1996).

————-, W.H. Beaver, and W. R. Landsman. The market valuation implications of net periodic pension cost. *Journal of Accounting and Economics* (1992), 27–62.

————-, W.H. Beaver, and C.H. Stinson. Supplemental data and the structure of thrift share prices. *The Accounting Review* (1991), 56–66.

————- and M.F. McNichols. Estimation and market valuation of environmental liabilities relating to Superfund sites. Supplement to the *Journal of Accounting Research* (1994), 177–209.

Beaver, W.H., C. Eger, S. Ryan, and M. Wolfson. Financial reporting, supplemental disclosures, and bank share prices. *Journal of Accounting Research* (1989), 157–178.

Bernard, V. L. Cross-sectional dependence and problems in inference in market-based accounting research. *Journal of Accounting Research* (1987), 1–48.

Bowen, R.M. Valuation of earnings components in the electric utility industry. *The Accounting Review* (1981), 1–22.

Chesher, A., and I. Jewitt. The bias of a heteroskedasticity consistent covariance matrix estimator. *Econometrica* (1987), 1217–1222.

Christie, A. On cross-sectional analysis in accounting research. *Journal of Accounting and Economics* (1987), 231–258.

Daley, L.A. The valuation of reported pension measures for firms sponsoring defined benefit plans. *The Accounting Review* (1984), 177–198.

Goldfeld, S.M., and R.E. Quandt. Some tests for homoscedasticity. *Journal of the American Statistical Association* (1965), 539–547.

Griliches, Z., and J.A. Hausman. Errors in variables in panel data. *Journal of Econometrics* (1986), 93–118.

Guenther, D., and M. Trombley. The LIFO reserve and the value of the firm: theory and empirical evidence. *Contemporary Accounting Research* (1994), 433–452.

Harris, T.S., and J.A. Ohlson. Accounting disclosures and the market's valuation of oil and gas properties. *The Accounting Review* (1987), 651–670.

Kothari, S.P., and J.L. Zimmerman. Price and return models. *Journal of Accounting and Economics* (1995), 155–192.

Kormendi, R., and R. Lipe. Earnings innovations, earnings persistence, and stock returns. *Journal of Business* (1987), 323–345.

Kuh, E., and J.R. Meyer. Correlation and regression estimates when the data are ratios, *Econometrica* (1955), 400–416.
Landsman, W. An empirical investigation of pension fund property rights. *The Accounting Review* (1986), 662–691.
———, and Magliolo, J. Cross-sectional capital market research and model specification. *The Accounting Review* (1988), 586–604.
Lev, B., and S. Sunder. Methodological issues in the use of financial ratios. *Journal of Accounting and Economics* (1979), 187–210.
Lustgarten, S. The impact of replacement cost disclosures on security prices: New evidence. *Journal of Accounting and Economics* (1982), 121–141.
Magliolo, J. Capital market analysis of reserve recognition accounting. *Journal of Accounting Research* (1986), 69–108.
Miller, M.H., and F. Modigliani. Some estimates of the cost of capital to the electric utility industry. *The American Economic Review* (1966), 333–391.
Ohlson, J.A. Accounting earnings, book value, and dividends: the theory of the clean surplus equation (part 1). Working paper. Columbia University, 1989.
Olsen, C. Valuation implications of SFAS no. 33 data for electric utility investors. Supplement to the *Journal of Accounting Research* (1985), 28–47.
Park, R. Estimation with heteroscedastic error terms. *Econometrica* (1966), 888.
Shevlin, T. The valuation of R&D firms with R&D limited partnerships. *The Accounting Review* (1991), 1–21.
Sougiannis, T. The accounting based valuation of corporate R&D. *The Accounting Review* (1994), 44–68.
White, H. A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica* (1980), 817–838.